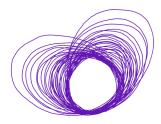
# orbitize Documentation

Sarah Blunt, Jason Wang, Henry Ngo, et al.

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Hello world! Welcome to the documentation for orbitize, a Python package for fitting orbits of directly imaged planets.

orbitize packages two back-end algorithms into a consistent API. It's written to be fast, extensible, and easy-to-use. The tutorials below will walk you through the code and introduce some technical stuff, but we suggest learning about the Orbits for the Impatient (OFTI) algorithm and MCMC algorithms (we use this one) before diving in. Our contributor guidelines document will point you to more useful resources.

orbitize is designed to meet the needs of the exoplanet imaging community, and we encourage community involvement. If you find a bug, want to request a feature, etc. please create an issue on GitHub.

orbitize is patterned after and inspired by radvel.

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#### **CHAPTER**

## **ONE**

## **ATTRIBUTION:**

- If you use orbitize in your work, please cite Blunt et al (2019).
- If you use the OFTI algorithm, please also cite Blunt et al (2017).
- If you use the Affine-invariant MCMC algorithm from emcee, please also cite Foreman-Mackey et al (2013).
- If you use the parallel-tempered Affine-invariant MCMC algorithm from ptemcee, please also cite Vousden et al (2016).
- If you use the Hipparcos intermediate astrometric data (IAD) fitting capability, please also cite Nielsen et al (2020) and van Leeuwen et al (2007).
- If you use Gaia data, please also cite Gaia Collaboration et al (2018; for DR2), or Gaia Collaboration et al (2021; for eDR3).

**CHAPTER** 

**TWO** 

## **USER GUIDE:**

## 2.1 Installation

#### 2.1.1 For Users

Parts of orbitize are written in C, so you'll need gcc (a C compiler) to install properly. Most Linux and Windows computers come with gcc built in, but Mac computers don't. If you haven't before, you'll need to download Xcode command line tools. There are several helpful guides online that teach you how to do this. Let us know if you have trouble!

orbitize is registered on pip, and works in Python>3.6. To install orbitize, first make sure you have the latest versions of numpy and cython installed. With pip, you can do this with the command:

```
$ pip install numpy cython --upgrade
```

Next, install orbitize:

```
$ pip install orbitize
```

We recommend installing and running orbitize in a conda virtual environment. Install anaconda or miniconda here, then see instructions here to learn more about conda virtual environments.

## 2.1.2 For Windows Users

Many of the packages that we use in orbitize were originally written for Linux or macOS. For that reason, we highly recommend installing the Windows Subsystem for Linux (WSL) which is an entire Linux development environment within Windows. See here for a handy getting started guide.

If you don't want to use WSL, there are a few extra steps you'll need to follow to get orbitize running:

1. There is a bug with the ptemcee installation that, as far as we know, only affects Windows users. To work around this, download ptemcee from its pypi page. Navigate to the root ptemcee folder, remove the README.md file, then install:

```
$ cd ptemcee
$ rm README.md
$ pip install . --upgrade
```

2. Some users have reported issues with installing curses. If this happens to you, you can install windows-curses which should work as a replacement.

```
$ pip install windows-curses
```

3. Finally, rebound is not compatible with windows, so you'll need to git clone orbitize, remove rebound from orbitize/requirements.txt, then install from the command line.

```
$ git clone https://github.com/sblunt/orbitize.git
$ cd orbitize
```

Open up orbitize/requirements.txt, remove rebound, and save.

```
$ pip install . --upgrade
```

## 2.1.3 For Developers

orbitize is actively being developed. The following method for installing orbitize will allow you to use it and make changes to it. After cloning the Git repository, run the following command in the top level of the repo:

```
$ pip install -r requirements.txt -e .
```

#### 2.1.4 Issues?

If you run into any issues installing orbitize, please create an issue on GitHub.

If you are specifically having difficulties using cython to install orbitize, we suggest first trying to install wheel, then installing all of the orbitize dependencies (listed in requirements.txt).

If that doesn't work, we suggest disabling compilation of the C-based Kepler module with the following alternative installation command:

```
$ pip install orbitize --install-option="--disable-cython"
```

#### 2.2 Tutorials

The following tutorials walk you through performing orbit fits with orbitize. To get started, read through "Formatting Input," "OFTI Introduction," and "MCMC Introduction." To learn more about the orbitize API, check out "Modifying Priors" and "Modifying MCMC Initial Positions." For an advanced plotting demo, see "Advanced Plotting," and to learn about the differences between OFTI and MCMC algorithms, we suggest "MCMC vs OFTI Comparison."

We also have a bunch of tutorials designed to introduce you to specific features of our code, listed below.

Many of these tutorials are also available as jupyter notebooks here.

If you find a bug, or if something is unclear, please create an issue on GitHub! We'd love any feedback on how to make orbitize more accessible.

A note about the tutorials: There are many ways to interact with the orbitize code base, and each person on our team uses the code differently. Each tutorial has a different author, and correspondingly a different style of using and explaining the code. If you are confused by part of one tutorial, we suggest looking at some of the others (and then contacting us if you are still confused).

## 2.2.1 Quick Start

This brief tutorial goes through the most minimal code you could write to do an orbit fit with orbitize!. It uses an input .csv that was placed on your computer when you installed orbitize!. The file lives here:

```
[1]: import orbitize

path_to_file = "{}GJ504.csv".format(orbitize.DATADIR)

print(path_to_file)

/home/sblunt/Projects/orbitize/orbitize/example_data/GJ504.csv
```

The input .csv file looks like this:

```
[2]: from orbitize import read_input
    read_input.read_file(path_to_file)
[2]: <Table length=7>
        epoch
                   object quant1 quant1_err ... quant12_corr quant_type instrument
       float64
                   int64 float64 float64
                                            . . .
                                                   float64
                                                                bytes5
                                                                            bytes5
                        1 2479.0
                                        16.0 ...
                                                                               defsp
          55645.95
                                                                    seppa
                                                          nan
          55702.89
                        1 2483.0
                                         8.0 ...
                                                          nan
                                                                    seppa
                                                                               defsp
         55785.015
                        1 2481.0
                                        33.0 ...
                                                          nan
                                                                               defsp
                                                                    seppa
                        1 2448.0
         55787.935
                                        24.0 ...
                                                          nan
                                                                    seppa
                                                                               defsp
    55985.19400184
                        1 2483.0
                                        15.0 ...
                                                          nan
                                                                   seppa
                                                                               defsp
    56029.11400323
                        1 2487.0
                                         8.0 ...
                                                                               defsp
                                                          nan
                                                                    seppa
    56072.30200459
                        1 2499.0
                                        26.0 ...
                                                          nan
                                                                    seppa
                                                                               defsp
```

## [3]: %matplotlib inline from orbitize import driver myDriver = driver.Driver( '{}/GJ504.csv'.format(orbitize.DATADIR), # data file 'OFTI', # choose from: ['OFTI', 'MCMC'] 1, # number of planets in system 1.22, # total mass [M\_sun] # system parallax [mas] 56.95, mass\_err=0.08, # mass error [M\_sun] plx\_err=0.26 # parallax error [mas] orbits = myDriver.sampler.run\_sampler(1000) # plot the results myResults = myDriver.sampler.results orbit\_figure = myResults.plot\_orbits( start\_mjd=myDriver.system.data\_table['epoch'][0] # minimum MJD for colorbar (choose\_ → first data epoch) )

```
WARNING: ErfaWarning: ERFA function "d2dtf" yielded 1 of "dubious year (Note 5)"
→[astropy._erfa.core]
WARNING: ErfaWarning: ERFA function "dtf2d" yielded 1 of "dubious year (Note 6)"
→[astropy._erfa.core]
WARNING: ErfaWarning: ERFA function "utctai" yielded 1 of "dubious year (Note 3)"
→[astropy._erfa.core]
WARNING: ErfaWarning: ERFA function "taiutc" yielded 1 of "dubious year (Note 4)"
→[astropy._erfa.core]
WARNING: ErfaWarning: ERFA function "dtf2d" yielded 8 of "dubious year (Note 6)".
→[astropy._erfa.core]
<Figure size 1008x432 with 0 Axes>
                                                                2800
                                                    2300
                                                                2700
                                                                2600
                                                              [mas]
                                                   2250
    5000
                                                              ď
                                                                2500
                                                                2400
                                                   2200
Mec [mas]
                                                                2300
                                                                            2015
                                                                                      2020
                                                                                                 2025
                                                   2150 H
                                                                 325
                                                   2100
                                                                320
                                                                 315
                                                    2050
   -10000
                                                                 310
             5000
                              -5000
                                      -10000
                                                                            2015
                                                                                      2020
                       Ó
                                                                                                 2025
                        ΔRA [mas]
                                                                                  Epoch
```

## 2.2.2 Formatting Input

Use orbitize.read\_input.read\_file() to read your astrometric data into orbitize. This method takes one argument, a string to the path of the file containing your input.

This method can read any file format supported by astropy.io.ascii.read(), including csv format. See the astropy does

There are two ways to provide input data to orbitize, either as observations or as an orbitize!-formatted input table.

## Option 1

You can provide your observations in one of the following valid sets of measurements using the corresponding column names:

- RA and DEC offsets [milliarcseconds], using column names raoff, raoff\_err, decoff, and decoff\_err; or
- sep [milliarcseconds] and PA [degrees East of NCP], using column names sep, sep\_err, pa, and pa\_err; or
- RV measurement [km/s] using column names rv and rv\_err.

Each row must also have a column for epoch and object. Epoch is the date of the observation, in MJD (JD-2400000.5). If this method thinks you have provided a date in JD, it will print a warning and attempt to convert to MJD. Objects are numbered with integers, where the primary/central object is 0.

You may mix and match these three valid measurement formats in the same input file. So, you can have some epochs with RA/DEC offsets and others in separation/PA measurements.

If you have, for example, one RV measurement of a star and three astrometric measurements of an orbiting planet, you should put 0 in the object column for the RV point, and 1 in the columns for the astrometric measurements.

This method will look for columns with the above labels in whatever file format you choose so if you encounter errors, be sure to double check the column labels in your input file.

Putting it all together, here an example of a valid .csv input file:

```
epoch,object,raoff,raoff_err,decoff,decoff_err,radec_corr,sep,sep_err,pa,pa_err,rv,rv_err
1234,1,0.010,0.005,0.50,0.05,0.025,,,,,
1235,1,,,,,1.0,0.005,89.0,0.1,,
1236,1,,,,,1.0,0.005,89.3,0.3,,
1237,0,,,,,,,,10,0.1
```

**Note:** Columns with no data can be omitted (e.g. if only separation and PA are given, the raoff, deoff, and rv columns can be excluded).

If more than one valid set is given (e.g. RV measurement and astrometric measurement taken at the same epoch), read\_file() will generate a separate output row for each valid set.

Whatever file format you choose, this method will read your input into an orbitize!-formatted input table. This is an astropy. Table.table object that looks like this (for the example input given above):

epoch o	bject int	quant1 q		quant2 q		quant12_corr float64	quant_type str5
1234.0 1235.0	1 1	0.01	0.005 0.005	0.5 89.0	0.05 0.1	0.025 nan	radec seppa
1236.0 1237.0	1	1.0 10.0	0.005 0.1	89.3 nan	0.3 nan	nan nan	seppa rv

where quant\_type is one of "radec", "seppa", or "rv".

If quant\_type is "radec" or "seppa", the units of quant are mas and degrees, if quant\_type is "rv", the units of quant are km/s.

#### **Covariances**

For RA/Dec and Sep/PA, you can optionally specify a correlation term. This is useful when your error ellipse is tilted with respect to the RA/Dec or Sep/PA. The correlation term is the Pearson correlation coefficient (), which corresponds to the normalized off diagonal term of the covariance matrix (C):

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{12} & C_{22} \end{bmatrix}.$$

Here C\_11 = quant1\_err^2 and C\_22 = quant2\_err^2 and C\_12 is the off diagonal term (note that by definition both off-diagonal terms of the covariance matrix are the same). Then,  $\rho = C_{12}/\sqrt{C_{11}C_{22}}$ . Essentially it is the covariance normalized by the variance. As such, -1 1. You can specify either as radec\_corr or seppa\_corr to include a correlation

in the errors. By definition, both are dimensionless, but one will correspond to RA/Dec and the other to Sep/PA. If no correlations are specified, it will assume the errors are uncorrelated (= 0). In many papers, the errors are assumed to be uncorrelated. An example of real world data that reports correlations is this GRAVITY paper where table 2 reports the correlation values and figure 4 shows how the error ellipses are tilted.

In the example above, we specify the first epoch has a positive correlation between the uncertainties in RA and Dec using the radec\_corr column in the input data. This gets translated into the quant12\_corr field in orbitize!-format. No correlations are specified for the other entries, and so we will assume those errors are uncorrelated. After this is specified, handling of the correlations will be done automatically when computing model likelihoods. There's nothing else you have to do after this step!

#### Option 2

Alternatively, you can also supply a data file with the columns already corresponding to the orbitize!-formatted input table (see above for column names). This may be useful if you are wanting to use the output of the write\_orbitize\_input method (e.g. using some input prepared by another orbitize! user).

**Note:** When providing data with columns in the orbitize format, there should be no empty cells. As in the example below, when quant2 is not applicable, the cell should contain nan.

#### 2.2.3 OFTI Introduction

by Isabel Angelo and Sarah Blunt (2018)

OFTI (Orbits For The Impatient) is an orbit-generating algorithm designed specifically to handle data covering short fractions of long-period exoplanets (Blunt et al. 2017). Here we go through steps of using OFTI within orbitize!

```
[1]: import orbitize
```

#### **Basic Orbit Generating**

Orbits are generated in OFTI through a Driver class within orbitize. Once we have imported this class:

```
[2]: import orbitize.driver
```

we can initialize a Driver object specific to our data:

```
[3]: myDriver = orbitize.driver.Driver('{}/GJ504.csv'.format(orbitize.DATADIR), # path to

data file

'OFTI', # name of algorithm for orbit-fitting

1, # number of secondary bodies in system

1.22, # total mass [M_sun]

56.95, # total parallax of system [mas]

mass_err=0.08, # mass error [M_sun]

plx_err=0.26) # parallax error [mas]
```

Because OFTI is an object class within orbitize, we can assign all of the OFTI attributes onto a variable (s). We can then generate orbits for s using a function called run\_sampler, a method of the OFTI class. The run\_sampler method takes in the desired number of accepted orbits as an input.

Here we use run OFTI to randomly generate orbits until 1000 are accepted:

```
[4]: s = myDriver.sampler
orbits = s.run_sampler(1000)
```

We have now generated 1000 possible orbits for our system. Here, orbits is a (1000 x 8) array, where each of the 1000 elements corresponds to a single orbit. An orbit is represented by 8 orbital elements.

Here is an example of what an accepted orbit looks like from orbitize:

To further inspect what each of the 8 elements in your orbit represents, you can view the system.param\_idx variable. This is a dictionary that tells you the indices of your orbit that correspond to semi-major axis (a), eccentricity (e), inclination (i), argument of periastron (aop), position angle of nodes (pan), and epoch of periastron passage (epp). The last two indices are the parallax and system mass, and the number following the parameter name indicates the number of the body in the system.

```
[6]: s.system.param_idx
[6]: {'sma1': 0,
    'ecc1': 1,
    'inc1': 2,
    'aop1': 3,
    'pan1': 4,
    'tau1': 5,
    'plx': 6,
    'mtot': 7}
```

#### **Plotting**

Now that we can generate possible orbits for our system, we want to plot the data to interpret our results. Here we will go through a brief overview on ways to visualize your data within orbitize. For a more detailed guide on data visualization capabilities within orbitize, see the Orbitize plotting tutorial.

#### Histogram

One way to visualize our results is through histograms of our computed orbital parameters. Our orbits are outputted from run\_sampler as an array of orbits, where each orbit is represented by a set of orbital elements:

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```
[3.89558225e+01, 3.71357464e-01, 2.94801131e+00, 3.08542398e+00, 4.77645562e+00, 7.15043369e-01, 5.72827098e+01, 1.05721164e+00], [8.19463988e+01, 1.45955646e-02, 2.11512811e+00, 4.52064036e+00, 4.44802306e+00, 9.32004660e-01, 5.72429430e+01, 1.35323242e+00]])
```

We can effectively view outputs from run\_sampler by creating a histogram of a given orbit element to see its distribution of possible values. Our system.param\_idx dictionary is useful here. We can use it to determine the index of a given orbit that corresponds to the orbital element we are interested in:

```
[8]: s.system.param_idx
[8]: {'sma1': 0,
    'ecc1': 1,
    'inc1': 2,
    'aop1': 3,
    'pan1': 4,
    'tau1': 5,
    'plx': 6,
    'mtot': 7}
```

If we want to plot the distribution of orbital semi-major axes (a) in our generated orbits, we would use the index dictionary s.system.param\_idx to index the semi-major axis element from each orbit:

```
[9]: sma = [x[s.system.param_idx['sma1']] for x in orbits]
     %matplotlib inline
     import matplotlib.pyplot as plt
     plt.hist(sma, bins=30)
     plt.xlabel('orbital semi-major axis [AU]')
     plt.ylabel('occurrence')
     plt.show()
        500
        400
      occurrence
        300
        200
        100
           0
                   100
                          200
                                 300
                                        400
                                                500
                                                       600
                                                              700
                            orbital semi-major axis [AU]
```

You can use this method to create histograms of any orbital element you are interested in:

(continued from previous page)

```
plt.figure(figsize=(10,3))
plt.subplot(131)
plt.hist(sma, bins=30)
plt.xlabel('orbital semi-major axis [AU]')
plt.ylabel('occurrence')
plt.subplot(132)
plt.hist(ecc, bins=30)
plt.xlabel('eccentricity [0,1]')
plt.ylabel('occurrence')
plt.subplot(133)
plt.hist(i, bins=30)
plt.xlabel('inclination angle [rad]')
plt.ylabel('occurrence')
plt.show()
   500
                                                                        70
                                      60
                                                                       60
   400
                                      50
                                                                       50
 occurrence
                                                                     occurrence
   300
                                      40
                                                                       40
                                      30
                                                                       30
   200
                                      20
                                                                       20
   100
                                      10
                                                                       10
     0
                                                                        0
                                                                                                 3.0
              200
                     400
                            600
                                         0.0
                                               0.2
                                                    0.4
                                                          0.6
                                                               0.8
                                                                              2.0
                                                                                        2.5
          orbital semi-major axis [AU]
                                                eccentricity [0,1]
                                                                               inclination angle [rad]
```

In addition to our orbits array, Orbitize also creates a Results class that contains built-in plotting capabilities for two types of plots: corner plots and orbit plots.

#### **Corner Plot**

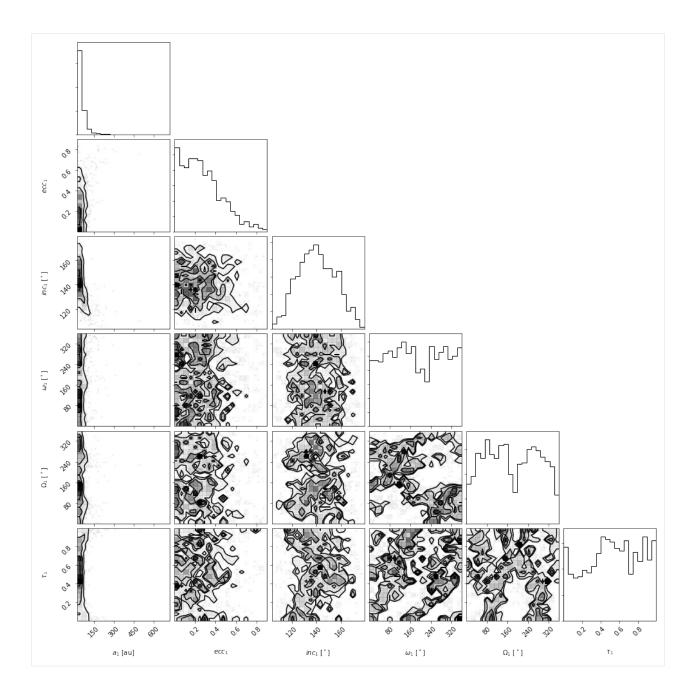
After generating the samples, the run\_sampler method also creates a Results object that can be accessed with s. results:

```
[11]: myResults = s.results
```

We can now create a corner plot using the function plot\_corner within the Results class. This function requires an input list of the parameters, in string format, that you wish to include in your corner plot. We can even plot all of the orbital parameters at once! As shown below:

```
[12]: corner_figure = myResults.plot_corner(param_list=['sma1', 'ecc1', 'inc1', 'aop1', 'pan1', 

→'tau1'])
```



## A Note about Convergence

Those of you with experience looking at corner plots will note that the result here does not look converged (i.e. we need more samples for our results to be statistically significant). Because this is a tutorial, we didn't want you to have to wait around for a while for the OFTI results to converge.

It's safe to say that OFTI should accept a minimum of 10,000 orbit for convergence. For pretty plots to go in publications, we recommend at least 1,000,000 accepted orbits.

#### **Orbit Plot**

What about if we want to see how the orbits look in the sky? Don't worry, the Results class has a command for that too! It's called plot\_orbits. We can create a simple orbit plot by running the command as follows:

```
[13]: epochs = myDriver.system.data_table['epoch']
      orbit_figure = myResults.plot_orbits(
           start_mjd=epochs[0] # Minimum MJD for colorbar (here we choose first data epoch)
      )
      WARNING: ErfaWarning: ERFA function "d2dtf" yielded 1 of "dubious year (Note 5)"
      →[astropy._erfa.core]
      WARNING: ErfaWarning: ERFA function "dtf2d" yielded 1 of "dubious year (Note 6)"
      →[astropy._erfa.core]
      WARNING: ErfaWarning: ERFA function "utctai" yielded 1 of "dubious year (Note 3)"
      →[astropy._erfa.core]
      WARNING: ErfaWarning: ERFA function "taiutc" yielded 1 of "dubious year (Note 4)"
      →[astropy._erfa.core]
      WARNING: ErfaWarning: ERFA function "dtf2d" yielded 8 of "dubious year (Note 6)"
      →[astropy._erfa.core]
      <Figure size 1008x432 with 0 Axes>
          10000
                                                                     2700
                                                         2120
                                                                     2600
                                                                     2500
                                                         2100
                                                                     2400
             0
                                                                     2300
       Mec [mas]
                                                         2080
                                                                                 2015
                                                                                           2020
                                                                                                     2025
                                                         2060
                                                                      325
         -10000
                                                                      320
                                                                    PA [ °]
                                                         2040
                                                                      315
                                                         2020
                                                                      310
         -20000
               20000
                         10000
                                     Ò
                                              -10000
                                                                                 2015
                                                                                           2020
                                                                                                     2025
                                                                                       Epoch
                              ΔRA [mas]
```

#### **Advanced OFTI and API Interaction**

We've seen how the run\_sampler command is the fastest way to generate orbits within OFTI. For users interested in what's going on under-the-hood, this part of the tutorial takes us each step of run\_sampler. Understanding the intermediate stages of orbit-fitting can allow for more customization that goes beyond Orbitize's default parameters.

We begin again by intializing a sampler object on which we can run OFTI:

```
[14]: myDriver = orbitize.driver.Driver('{}/GJ504.csv'.format(orbitize.DATADIR), # path to

data file

'OFTI', # name of algorith for orbit-fitting

1, # number of secondary bodies in system

(continues on next page)
```

(continued from previous page)

```
1.22, # total mass [M_sun]
56.95, # total parallax of system [mas]
mass_err=0.08, # mass error [M_sun]
plx_err=0.26) # parallax error [mas]
```

```
[15]: s = myDriver.sampler
```

In orbitize, the first thing that OFTI does is prepare an initial set of possible orbits for our object through a function called prepare\_samples, which takes in the number of orbits to generate as an input. For example, we can generate 100,000 orbits as follows:

```
[16]: samples = s.prepare_samples(100000)
```

Here, samples is an array of randomly generated orbits that have been scaled-and-rotated to fit our astrometric observations. The first and second dimension of this array are the number of orbital elements and total orbits generated, respectively. In other words, each element in samples represents the value of a particular orbital element for each generated orbit:

```
[17]: print('samples: ', samples.shape)
    print('first element of samples: ', samples[0].shape)

samples: (8, 100000)
    first element of samples: (100000,)
```

Once our initial set of orbits is generated, the orbits are vetted for likelihood in a function called reject. This function computes the probability of an orbit based on its associated chi squared. It then rejects orbits with lower likelihoods and accepts the orbits that are more probable. The output of this function is an array of possible orbits for our input system.

```
[18]: orbits, lnlikes = s.reject(samples)
```

Our orbits array represents the final orbits that are output by OFTI. Each element in this array contains the 8 orbital elements that are computed by orbitize:

```
[19]: orbits.shape
[19]: (1, 8)
```

We can synthesize this sequence with the run\_sampler() command, which runs through the steps above until the input number of orbits has been accepted. Additionally, we can specify the number of orbits generated by prepare\_samples each time the sequence is initiated with an argument called num\_samples. Higher values for num\_samples will output more accepted orbits, but may take longer to run since all initially prepared orbits will be run through the rejection step.

```
[20]: orbits = s.run_sampler(100, num_samples=1000)
```

#### **Saving and Loading Results**

Finally, we can save our generated orbits in a file that can be easily read for future use and analysis. Here we will walk through the steps of saving a set of orbits to a file in hdf5 format. The easiest way to do this is using orbitize. Results.save\_results():

```
[21]: s.results.save_results('orbits.hdf5')
```

Now when you are ready to use your orbits data, it is easily accessible through the file we've created. One way to do this is to load the data into a new results object; in this way you can make use of the functions that we learn before, like plot\_corner and plot\_orbits. To do this, use the results module:

Alternatively, you can directly access the saved data using the h5py module:

And now we can easily work with the saved orbits that were generated by orbitize! Find out more about generating orbits in orbitize! with tutorials here.

## 2.2.4 MCMC Introduction

by Jason Wang and Henry Ngo (2018)

Here, we will explain how to sample an orbit posterior using MCMC techniques. MCMC samplers take some time to fully converge on the complex posterior, but should be able to explore all posteriors in roughly the same amount of time (unlike OFTI). We will use the parallel-tempered version of the Affine-invariant sample from the ptemcee package, as the parallel tempering helps the walkers get out of local minima. Parallel-tempering can be disabled by setting the number of temperatures to 1, and will revert back to using the regular ensemble sampler from emcee.

#### Read in Data and Set up Sampler

We use orbitize.driver.Driver to streamline the processes of reading in data, setting up the two-body interaction, and setting up the MCMC sampler.

When setting up the sampler, we need to decide how many temperatures and how many walkers per temperature to use. Increasing the number of temperatures further ensures your walkers will explore all of parameter space and will not get stuck in local minima. Increasing the number of walkers gives you more samples to use, and, for the Affine-invariant sampler, a minimum number is required for good convergence. Of course, the tradeoff is that more samplers means more computation time. We find 20 temperatures and 1000 walkers to be reliable for convergence. Since this is a tutorial meant to be run quickly, we use fewer walkers and temperatures here.

Note that we will only use the samples from the lowest-temperature walkers. We also assume that our astrometric measurements follow a Gaussian distribution.

orbitize can also fit for the total mass of the system and system parallax, including marginalizing over the uncertainties in those parameters.

```
[2]: import numpy as np
    import orbitize
    from orbitize import driver
    import multiprocessing as mp
    filename = "{}/GJ504.csv".format(orbitize.DATADIR)
    # system parameters
    num_secondary_bodies = 1
    total_mass = 1.75 # [Msol]
    plx = 51.44 \# [mas]
    mass_err = 0.05 # [Msol]
    plx_err = 0.12 # [mas]
    # MCMC parameters
    num\_temps = 5
    num_walkers = 20
    num_threads = 2  # or a different number if you prefer, mp.cpu_count() for example
    my_driver = driver.Driver(
        filename.
         "MCMC",
        num_secondary_bodies,
        total_mass,
        plx.
        mass_err=mass_err,
        plx_err=plx_err,
        mcmc_kwargs={
             "num_temps": num_temps,
             "num_walkers": num_walkers,
             "num_threads": num_threads,
        },
    )
```

#### **Running the MCMC Sampler**

We need to pick how many steps the MCMC sampler should sample. Additionally, because the samples are correlated, we often only save every nth sample. This helps when we run a lot of samples, because saving all the samples requires too much disk space and many samples are unnecessary because they are correlated.

```
[3]: total_orbits = 6000  # number of steps x number of walkers (at lowest temperature)
    burn_steps = 10  # steps to burn in per walker
    thin = 2 # only save every 2nd step
    my_driver.sampler.run_sampler(total_orbits, burn_steps=burn_steps, thin=thin)
    /home/sblunt/Projects/orbitize/orbitize/priors.py:354: RuntimeWarning: invalid value_
    →encountered in log
      lnprob = -np.log((element_array*normalizer))
    /home/sblunt/Projects/orbitize/orbitize/priors.py:463: RuntimeWarning: invalid value_
    →encountered in log
      lnprob = np.log(np.sin(element_array)/normalization)
    Starting Burn in
    /home/sblunt/Projects/orbitize/orbitize/priors.py:354: RuntimeWarning: invalid value_
    →encountered in log
      lnprob = -np.log((element_array*normalizer))
    /home/sblunt/Projects/orbitize/orbitize/priors.py:463: RuntimeWarning: invalid value_
    →encountered in log
      lnprob = np.log(np.sin(element_array)/normalization)
    10/10 steps of burn-in complete
    Burn in complete. Sampling posterior now.
    300/300 steps completed
    Run complete
```

After completing the samples, the 'run\_sampler' method also creates a 'Results' object that can be accessed with 'my\_sampler.results'.

#### **MCMC Convergence**

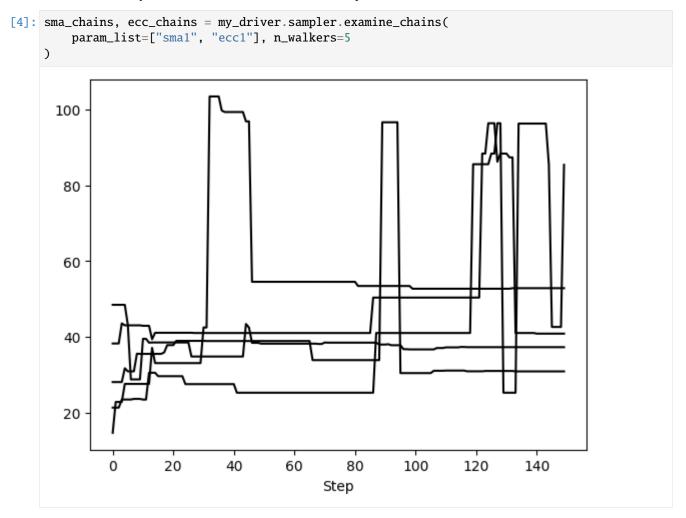
We want our walkers to be "converged" before they accurately sample the full posterior of our fitted parameters. Formal proofs of MCMC convergence are difficult or impossible in some cases. Many tests of convergence are necesary but not sufficient proofs of convergence. Here, we provide some convience functions to help assess convergence, but we caution they are not foolproof. A more detailed description of convergence analysis for affine-invariant samples (which are the ones used in orbitize!) is available *in the* "emcee" docs <a href="https://emcee.readthedocs.io/en/stable/tutorials/autocorr/">" \_\_\_\_."

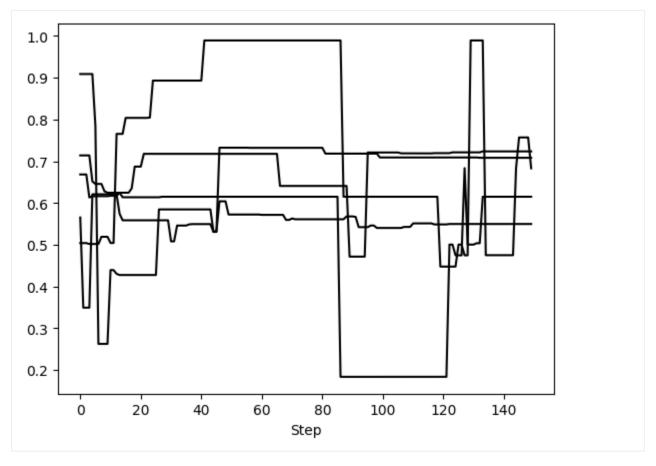
One of the primary ways we assess convergence for orbit fits for MCMC is the visual inspection of the chains. This is done by looking at some key parameters such as semi-major axis and eccentricity and plotting the value sampled for each chain as a function of step number. At the beginning, the ensemble of walkers is going to expand/contract/wiggle around to figure out where the posterior space is. Eventually, the walkers will appear to reach some "thermal equilibrium" beyond which the values sampled by the ensemble of walkers appear to not change with time. Below is how to use some built-in diagnostic functions for this.

### **Diagnostic Functions**

The Sampler object also has two convenience functions to examine and modify the walkers in order to diagnose MCMC performance. Note that in this example we have not run things for long enough with enough walkers to actually see convergence, so this is merely a demo of the API.

First, we can examine 5 randomly selected walkers for two parameters: semimajor axis and eccentricity. We expect 150 steps per walker since there were 6,000 orbits requested with 20 walkers, so that's 300 orbits per walker. However, we have thinned by a factor of 2, so there are 150 saved steps.





This method returns one matplotlib Figure object for each parameter. If no param\_list given, all parameters are returned. Here, we told it to plot 5 randomly selected walkers but we could have specified exactly which walkers with the walker\_list keyword. The step\_range keyword also determines which steps in the chain are plotted (when nothing is given, the default is to plot all steps). We can also have these plots automatically generate if we called run\_sampler with examine\_chains=True.

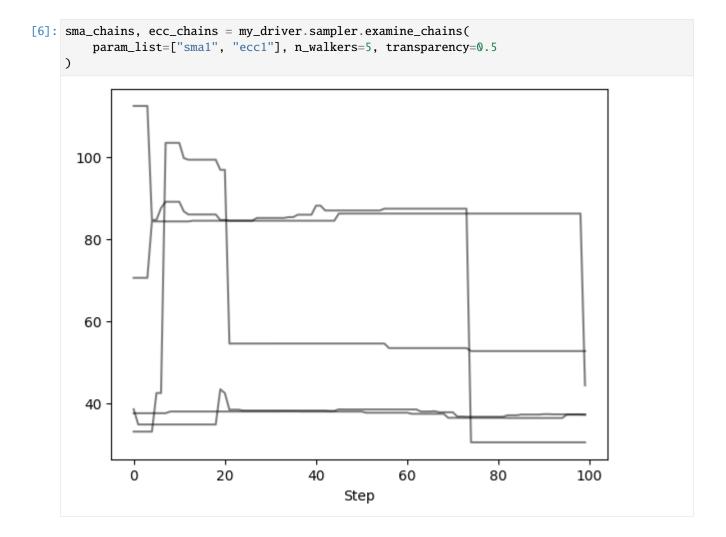
Note that this is just a convenience function. It is possible to recreate these chains from reshaping the posterior samples and selecting the correct entries.

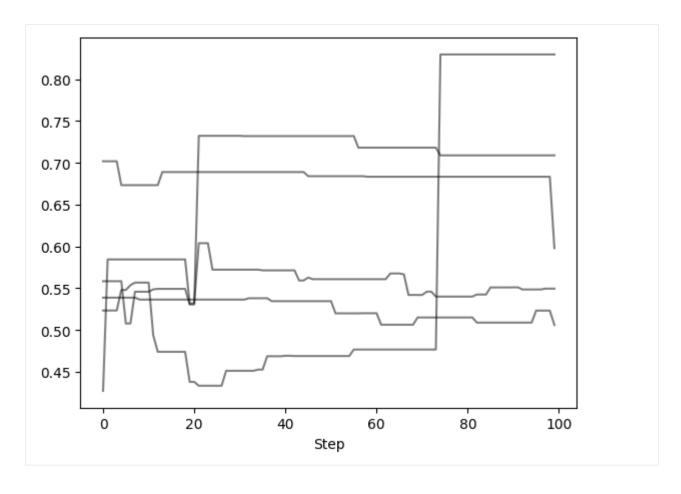
The second diagnostic tool is the chop\_chains, which allows us to remove entries from the beginning and/or end of a chain. This updates the corresponding Results object stored in sampler (in this case, my\_driver.sampler. results). The burn parameter specifies the number of steps to remove from the beginning (i.e. to add a burn-in to your chain) and the trim parameter specifies the number of steps to remove from the end. If only one parameter is given, it is assumed to be a burn value. If trim is not zero, the sampler object is also updated so that the current position (sampler.curr\_pos) matches the new end point. This allows us to continue MCMC runs at the correct position, even if we have removed the last few steps of the chain.

Let's remove the first and last 25 steps, leaving 100 orbits (or steps) per walker

[5]: my\_driver.sampler.chop\_chains(burn=25, trim=25)
Chains successfully chopped. Results object updated.

Now we can examine the chains again to verify what we did. Note that the number of steps removed from either end of the chain is uniform across all walkers.





#### **Autocorrelation Time Estimation**

We can use the emcee package to estimate the autocorrelation time from our chains.

The integrated autocorrelation time  $(\tau)$  for each parameter is returned, with an estimate of  $\sim$  10-15 steps needed for the chain to "forget" where it started. Notice an AutocorrError is returned since we have not run our chain long enough. Therefore, we should treat the integrated autocorrelation time as a lower bound, and run the MCMC for more steps. Here is a tutorial from emcee that estimates a more accurate integrated autocorrelation time when the chains are properly converged.

```
flatchain = my_driver.sampler.post
total_samples, n_params = flatchain.shape
n_steps = int(total_samples / num_walkers)
chn = flatchain.reshape(num_walkers, n_steps, n_params)
# For emcee, reshape to (n_steps, num_walkers, n_params)
chn = np.transpose(chn, axes=(1, 0, 2))

try:
    tau = emcee.autocorr.integrated_time(chn)
except Exception as e:
    print("Exception was raised! The error message is: \n \n{}".format(e))
```

```
Exception was raised! The error message is:

The chain is shorter than 50 times the integrated autocorrelation time for 8 parameter(s). Use this estimate with caution and run a longer chain!

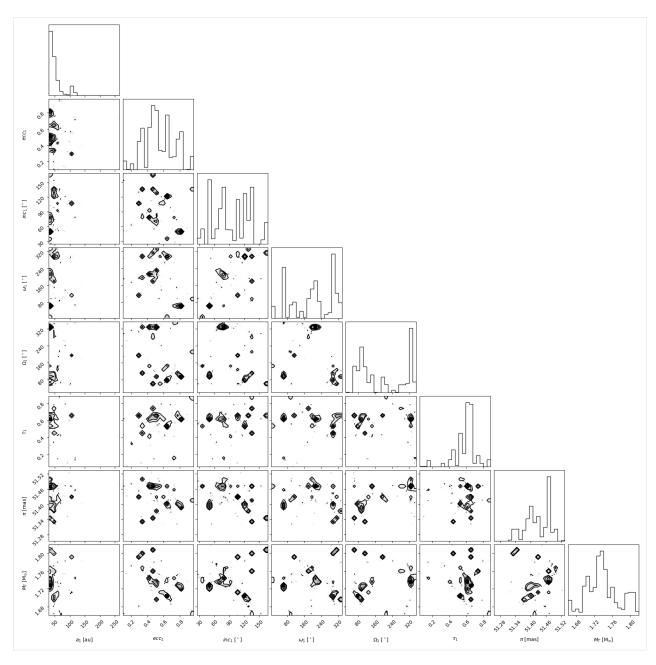
N/50 = 3;

tau: [11.72814516 12.47675593 13.46492791 13.94748748 12.62859548 12.64879897 12.94860228 13.10522437]
```

#### **Plotting Basics**

We will make some basic plots to visualize the samples in 'my\_driver.sampler.results'. Orbitize currently has two basic plotting functions which return matplotlib Figure objects. First, we can make a corner plot (also known as triangle plot, scatterplot matrix, or pairs plot) to visualize correlations between pairs of orbit parameters:

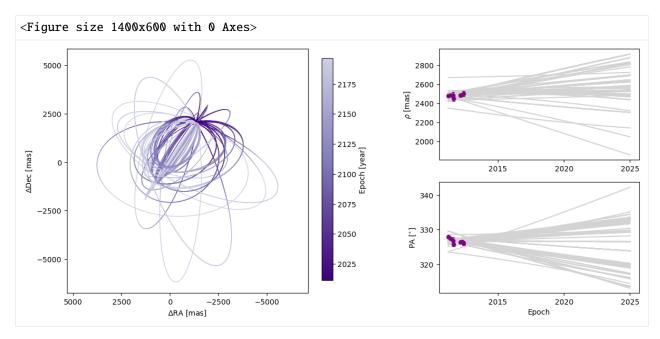
```
[7]: corner_plot_fig = (
         my_driver.sampler.results.plot_corner()
) # Creates a corner plot and returns Figure object
corner_plot_fig.savefig(
         "my_corner_plot.png"
) # This is matplotlib.figure.Figure.savefig()
```



Next, we can plot a visualization of a selection of orbits sampled by our sampler. By default, the first epoch plotted is the year 2000 and 100 sampled orbits are displayed.

```
[8]: epochs = my_driver.system.data_table["epoch"]

orbit_plot_fig = my_driver.sampler.results.plot_orbits(
    object_to_plot=1, # Plot orbits for the first (and only, in this case) companion
    num_orbits_to_plot=100, # Will plot 100 randomly selected orbits of this companion
    start_mjd=epochs[0], # Minimum MJD for colorbar (here we choose first data epoch)
)
orbit_plot_fig.savefig(
    "my_orbit_plot.png"
) # This is matplotlib.figure.Figure.savefig()
```



For more advanced plotting options and suggestions on what to do with the returned matplotlib Figure objects, see the dedicated Plotting tutorial.

#### **Saving and Loading Results**

We will save the results in the HDF5 format. It will save two datasets: 'post' which will contain the posterior (the chains of the lowest temperature walkers) and 'lnlike' which has the corresponding probabilities. In addition, it saves 'sampler\_name' and the orbitize version number as attributes of the HDF5 root group, and the orbitize. system.System object used to generate the orbit fit.

Saving sampler results is a good idea when we want to analyze the results in a different script or when we want to save the output of a long MCMC run to avoid having to re-run it in the future. We can then load the saved results into a new blank results object.

```
[10]: from orbitize import results
loaded_results = results.Results() # Create blank results object for loading
loaded_results.load_results(hdf5_filename)
```

Instead of loading results into an orbitize.results.Results object, we can also directly access the saved data using the 'h5py' python module.

```
[11]: import h5py

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```

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```
filename = "my_posterior.hdf5"
hf = h5py.File(filename, "r") # Opens file for reading
# Load up each dataset from hdf5 file
sampler_name = str(hf.attrs["sampler_name"])
post = np.array(hf.get("post"))
lnlike = np.array(hf.get("lnlike"))
hf.close() # Don't forget to close the file
```

## 2.2.5 Modifying Priors

by Sarah Blunt (2018)

Most often, you will use the Driver class to interact with orbitize. This class automatically reads your input file, creates all of the orbitize objects you need to run an orbit fit, and allows you to run the orbit fit. See the introductory OFTI and MCMC tutorials for examples of working with this class.

However, sometimes you will want to work with the underlying methods directly. Doing this gives you control over the functionality Driver executes automatically, and allows you more flexibility.

Modifying priors is an example of something you might want to use the underlying API for. This tutorial walks you through how to do that.

Goals of this tutorial: - Learn to modify priors in orbitize - Learn how to fix a parameter at a specific value - Learn about the structure of the orbitize code base

```
[1]: import numpy as np
  from matplotlib import pyplot as plt
  import orbitize
  from orbitize import read_input, system, priors, sampler
```

#### **Read in Data**

First, let's read in our data table. This is accomplished with orbitize.read\_input:

```
[2]: data_table = read_input.read_file('{}/GJ504.csv'.format(orbitize.DATADIR))
    print(data_table)
        epoch
                   object quant1 quant1_err ... quant12_corr quant_type instrument
           ------ ----- ----- ------ ,,, ------- ----
                        1 2479.0
                                       16.0 ...
                                                                             defsp
          55645.95
                                                         nan
                                                                  seppa
          55702.89
                        1 2483.0
                                       8.0 ...
                                                         nan
                                                                  seppa
                                                                             defsp
         55785.015
                        1 2481.0
                                       33.0 ...
                                                         nan
                                                                  seppa
                                                                             defsp
         55787.935
                        1 2448.0
                                       24.0 ...
                                                                             defsp
                                                         nan
                                                                  seppa
    55985.19400184
                        1 2483.0
                                       15.0 ...
                                                                             defsp
                                                                  seppa
                                                         nan
                        1 2487.0
                                                                             defsp
    56029.11400323
                                       8.0 ...
                                                         nan
                                                                  seppa
    56072.30200459
                        1 2499.0
                                       26.0 ...
                                                                             defsp
                                                         nan
                                                                  seppa
```

#### **Initialize System Object**

Next, we initialize an orbitize.system.System object. This object stores information about the system you're fitting, such as your data, the total mass, and the parallax.

```
[3]: # number of secondary bodies in system
    num_planets = 1

# total mass & error [msol]
    total_mass = 1.22
    mass_err = 0.08

# parallax & error[mas]
    plx = 56.95
    plx_err = 0

sys = system.System(
        num_planets, data_table, total_mass,
        plx, mass_err=mass_err, plx_err=plx_err
)
```

The System object has a few handy attributes to help you keep track of your fitting parameters. System.labels is a list of the names of your fit parameters, and System.sys\_priors is a list of the priors on each parameter. Notice that the "prior" on parallax (plx) is just a float. That's because we fixed this parameter at the printed value by specifying that plx\_err=0.

Finally, System.param\_idx is a dictionary that maps the parameter names from System.labels to their indices in System.sys\_priors.

```
[4]: print(sys.labels)
    print(sys.sys_priors)
    print(sys.param_idx)

# alias for convenience
lab = sys.param_idx

['sma1', 'ecc1', 'inc1', 'aop1', 'pan1', 'tau1', 'plx', 'mtot']
[Log Uniform, Uniform, Sine, Uniform, Uniform, 56.95, Gaussian]
    {'sma1': 0, 'ecc1': 1, 'inc1': 2, 'aop1': 3, 'pan1': 4, 'tau1': 5, 'plx': 6, 'mtot': 7}
```

#### **Explore & Modify Priors**

Priors in orbitize are Python objects. You can view an exhaustive list here. Let's print out the attributes of some of our priors:

Check out the priors documentation (linked above) for more info about the attributes of each of these priors.

Now that we understand how priors are represented and where they are stored, we can modify them! Here's an example of changing the prior on eccentricity from the current uniform prior to a Gaussian prior:

```
[6]: mu = 0.2
    sigma = 0.05

sys.sys_priors[lab['ecc1']] = priors.GaussianPrior(mu, sigma)

print(sys.labels)
    print(sys.sys_priors)
    print(vars(sys.sys_priors[lab['ecc1']]))

['sma1', 'ecc1', 'inc1', 'aop1', 'pan1', 'tau1', 'plx', 'mtot']
    [Log Uniform, Gaussian, Sine, Uniform, Uniform, 56.95, Gaussian]
    {'mu': 0.2, 'sigma': 0.05, 'no_negatives': True}
```

Let's do one more example. Say we want to fix the inclination to a particular value (i.e. not allow it to vary in the fit at all), perhaps the known inclination value of a disk in the system. We can do that as follows:

```
[7]: sys.sys_priors[lab['inc1']] = 2.5

print(sys.labels)
print(sys.sys_priors)
print('Inclination "prior:" {}'.format(sys.sys_priors[sys.param_idx['inc1']]))
print('Eccentricity prior: {}'.format(sys.sys_priors[sys.param_idx['ecc1']]))

['sma1', 'ecc1', 'inc1', 'aop1', 'pan1', 'tau1', 'plx', 'mtot']
[Log Uniform, Gaussian, 2.5, Uniform, Uniform, Uniform, 56.95, Gaussian]
Inclination "prior:" 2.5
Eccentricity prior: Gaussian
```

#### **Run OFTI**

All right! We're in business. To finish up, I'll demonstrate how to run an orbit fit with our modified System object, first with OFTI, then with MCMC.

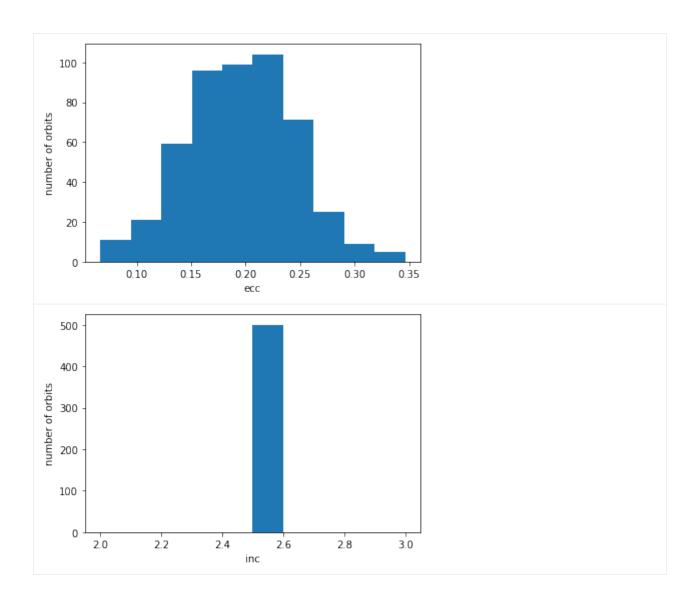
```
[8]: ofti_sampler = sampler.OFTI(sys)

# number of orbits to accept
n_orbs = 500

_ = ofti_sampler.run_sampler(n_orbs)

plt.figure()
accepted_eccentricities = ofti_sampler.results.post[:, lab['ecc1']]
plt.hist(accepted_eccentricities)
plt.xlabel('ecc'); plt.ylabel('number of orbits')

plt.figure()
accepted_inclinations = ofti_sampler.results.post[:, lab['inc1']]
plt.hist(accepted_inclinations)
plt.xlabel('inc'); plt.ylabel('number of orbits')
[8]: Text(0, 0.5, 'number of orbits')
```



## **Run MCMC**

```
[9]: # number of temperatures & walkers for MCMC
num_temps = 3
num_walkers = 50

# number of steps to take
n_orbs = 500

mcmc_sampler = sampler.MCMC(sys, num_temps, num_walkers)

# number of orbits to accept
n_orbs = 500

_ = mcmc_sampler.run_sampler(n_orbs)
```

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```
accepted_eccentricities = mcmc_sampler.results.post[:, lab['ecc1']]
     plt.hist(accepted_eccentricities)
     plt.xlabel('ecc'); plt.ylabel('number of orbits')
     Starting Burn in
     Burn in complete. Sampling posterior now.
     /home/sblunt/Projects/orbitize/orbitize/priors.py:354: RuntimeWarning: invalid value_
     →encountered in log
       lnprob = -np.log((element_array*normalizer))
     10/10 steps completed
     Run complete
[9]: Text(0, 0.5, 'number of orbits')
        175
        150
        125
     number of orbits
        100
```

## 2.2.6 Advanced Plotting

0.10

0.15

0.20

0.25

0.30

by Henry Ngo (2018)

75

50

25

0

The results.py module contains several plotting functions to visualize the results of your orbitize orbit fit. Basic uses of these functions are covered in the OFTI and MCMC tutorials. Here, we will examine these plotting functions more deeply. This tutorial will be updated as more features are added to orbitize.

0.35

0.40

#### 1. Test orbit generation with OFTI

In order to have sample data for this tutorial, we will use OFTI to generate some orbits for a published dataset on the GJ 504 system. The following code block is from the OFTI Tutorial, with 10000 orbits generated. Please see that tutorial for details.

Note: If you have already run this tutorial and saved the computed orbits, you may skip to Section 3 and load up your previously computed orbits instead of running this block below.

```
[2]: import orbitize.driver
                                                                                                   (continues on next page)
```

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```
myDriver = orbitize.driver.Driver(
    '{}/GJ504.csv'.format(orbitize.DATADIR), # relative or absolute path to data file
    'OFTI', # name of algorithm for orbit-fitting
    1, # number of secondary bodies in system
    1.22, # total mass [M_sun]
    56.95, # parallax of system [mas]
    mass_err=0.08, # mass error [M_sun]
    plx_err=0.26 # parallax error [mas]
)
s = myDriver.sampler
orbits = s.run_sampler(1000)
1000/1000 orbits found
```

#### 2. Accessing a Results object with computed orbits

After computing your orbits from either OFTI or MCMC, they are accessible as a Results object for further analysis and plotting. This object is an attribute of s, the sampler object defined above.

```
[3]: myResults = s.results # array of MxN array of orbital parameters (M orbits with N_{-} \rightarrow parameters per orbit)
```

It is also useful to save this Results object to a file if we want to load up the same data later without re-computing the orbits.

```
[4]: myResults.save_results('plotting_tutorial_GJ504_results.hdf5')
```

For more information on the Results object, see below.

```
[5]: myResults?
```

```
Type:
             Results
String form: <orbitize.results.Results object at 0x7fbbbf884be0>
             ~/Documents/GitHub/orbitize/orbitize/results.py
File:
Docstring:
A class to store accepted orbital configurations from the sampler
Args:
    system (orbitize.system.System): System object used to do the fit.
    sampler_name (string): name of sampler class that generated these results
        (default: None).
   post (np.array of float): MxN array of orbital parameters
        (posterior output from orbit-fitting process), where M is the
       number of orbits generated, and N is the number of varying orbital
        parameters in the fit (default: None).
    Inlike (np.array of float): M array of log-likelihoods corresponding to
        the orbits described in ``post`` (default: None).
   version_number (str): version of orbitize that produced these results.
   data (astropy.table.Table): output from ``orbitize.read_input.read_file()``
    curr_pos (np.array of float): for MCMC only. A multi-D array of the
        current walker positions that is used for restarting a MCMC sampler.
```

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```
Written: Henry Ngo, Sarah Blunt, 2018

API Update: Sarah Blunt, 2021
```

Note that you can also add more computed orbits to a results object with myResults.add\_samples():

```
[6]: myResults.add_samples?
    Signature: myResults.add_samples(orbital_params, lnlikes, curr_pos=None)
    Add accepted orbits, their likelihoods, and the orbitize version number
    to the results
    Args:
        orbital_params (np.array): add sets of orbital params (could be multiple)
            to results
        lnlike (np.array): add corresponding lnlike values to results
        curr_pos (np.array of float): for MCMC only. A multi-D array of the
            current walker positions
    Written: Henry Ngo, 2018
    API Update: Sarah Blunt, 2021
    File:
               ~/Documents/GitHub/orbitize/orbitize/results.py
    Type:
               method
```

### 3. (Optional) Load up saved results object

If you are skipping the generation of all orbits because you would rather load from a file that saved the Results object generated above, then execute this block to load it up. Otherwise, skip this block (however, nothing bad will happen if you run it even if you generated orbits above).

```
[7]: import orbitize.results
   if 'myResults' in locals():
        del myResults # delete existing Results object
   myResults = orbitize.results.Results() # create empty Results object
   myResults.load_results('plotting_tutorial_GJ504_results.hdf5') # load from file
```

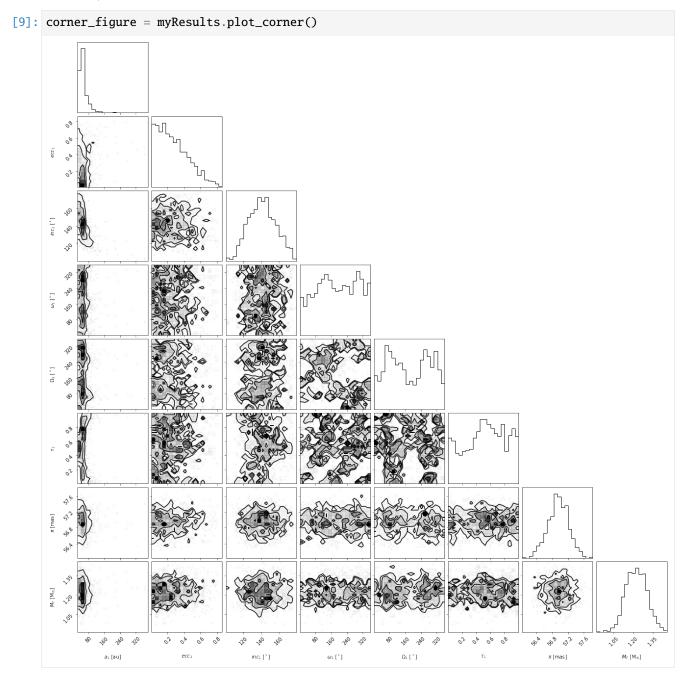
# 4. Using our Results object to make plots

In this tutorial, we'll work with two plotting functions: plot\_corner() makes a corner plot and plot\_orbits() displays some or all of the computed orbits. Both plotting functions return matplotlib.pyplot.figure objects, which can be displayed, further manipulated with matplotlib.pyplot functions, and saved.

```
[8]: %matplotlib inline
import matplotlib.pyplot as plt
```

# 4.1 Corner plots

This function is a wrapper for corner.py and creates a display of the 2-D covariances between each pair of parameters as well as histograms for each parameter. These plots are known as "corner plots", "pairs plots", and "scatterplot matrices", as well as other names.

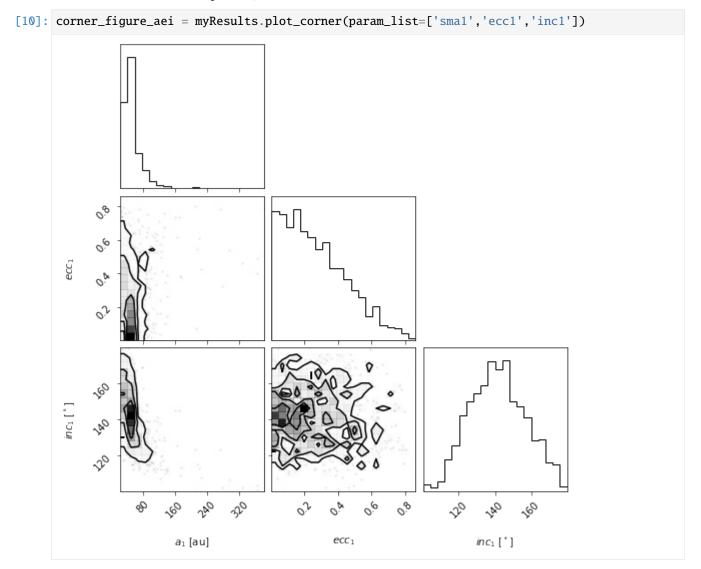


# Choosing which parameters to plot

Sometimes, the full plot with all parameters is not what we want. Let's use the param\_list keyword argument to plot only semimajor axis, eccentricity and inclination. Here are the possible string labels for this fit that you can enter for param\_list and the corresponding orbit fit parameter:

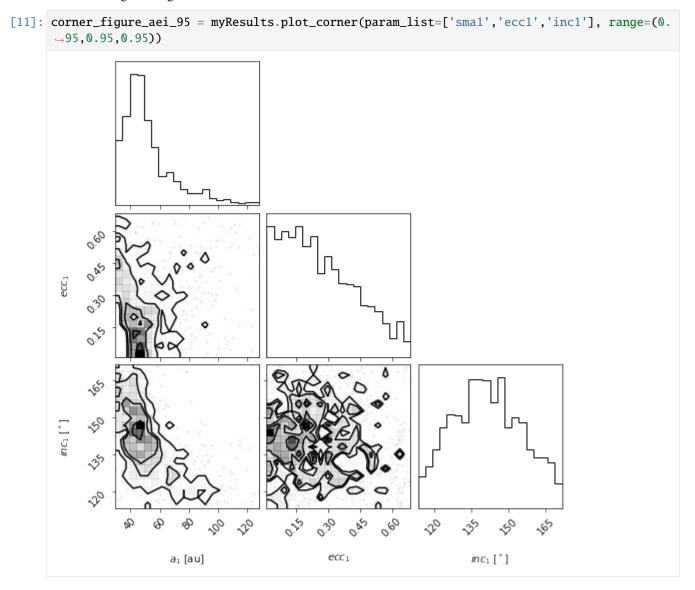
Label	Parameter name
sma1	semimajor axis
ecc1	eccentricity
inc1	inclination
aop1	argument of periastron
pan1	position angle of nodes (aka longitude of ascending node)
tau1	epoch of periastron passage (expressed as a fraction of orbital period past a specified offset)
mtot	system mass
plx	system parallax

Note: for labels with numbers, the number corresponds to the companion (sma1 is the first object's semimajor axis, sma2 would be the second object, etc)



## Limiting which samples to display

By picking out the panels we show, the plot can be easier to read. But in this case, we see that the plotted x-range on semimajor axis does show the posterior very well. This function will pass on all corner.corner() keywords as well. For example, we can use corner's range keyword argument to limit the panels to only display 95% of the samples to avoid showing the long tails in the distribution.



For other keywords you can pass to corner, see the corner.py API.

## Making single variable histogram plots

One use of the param\_list keyword is to just plot the histogram for the distribution of one single orbit fit parameter. We can do this by just providing one single parameter.

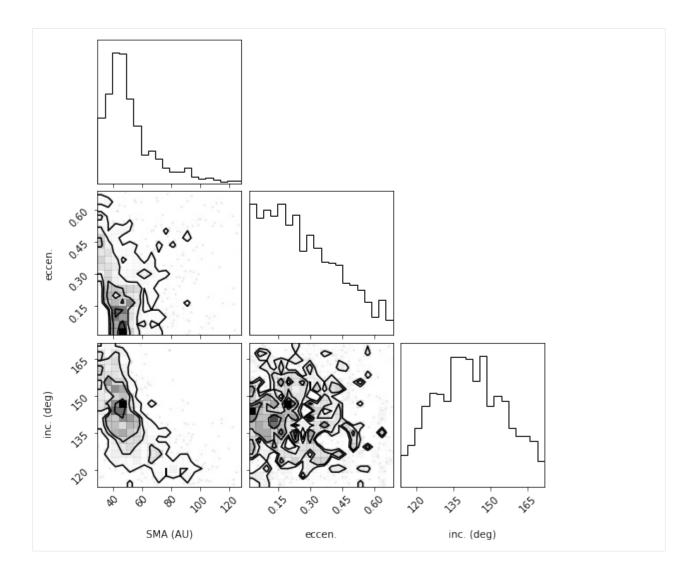


#### **Axis label customization**

The axis labels seen on the above plots are the default labels that orbitize passes to corner.py to make these plots. We can override these defaults by passing our own labels through the labels keyword parameter as per the corner.py API.

Note that the length of the list of labels should match the number of parameters plotted.

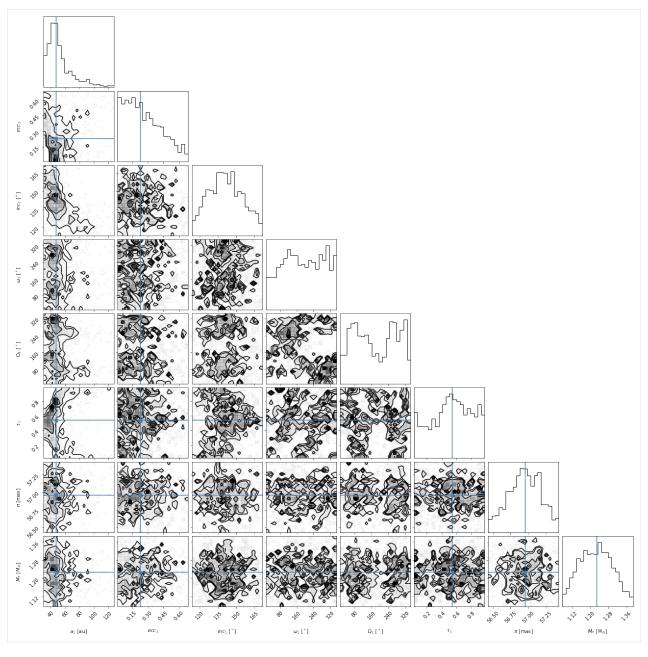
```
[13]: # Corner plot with alternate labels
corner_figure_aei_95_labels = myResults.plot_corner(
    param_list=['sma1','ecc1','inc1'],
    range=(0.95,0.95,0.95),
    labels=('SMA (AU)', 'eccen.', 'inc. (deg)')
)
```



## Overplotting best fit ("truth") values

One feature of corner.py is to overplot the contours and histograms with a so-called "truth" value, which we can use for many purposes. For example, if we are sampling from a known distribution, we can use it to plot the true value to compare with our samples. Or, we can use it to mark the location of the mean or median of the distribution (the histogram and contours make it easy to pick out the most likely value or peaks of the distribution but maybe not the mean or median). Here is an example of overplotting the median on top of the distribution for the full corner plot.

```
import numpy as np
median_values = np.median(myResults.post,axis=0) # Compute median of each parameter
range_values = np.ones_like(median_values)*0.95 # Plot only 95% range for each parameter
corner_figure_median_95 = myResults.plot_corner(
    range=range_values,
    truths=median_values
)
```

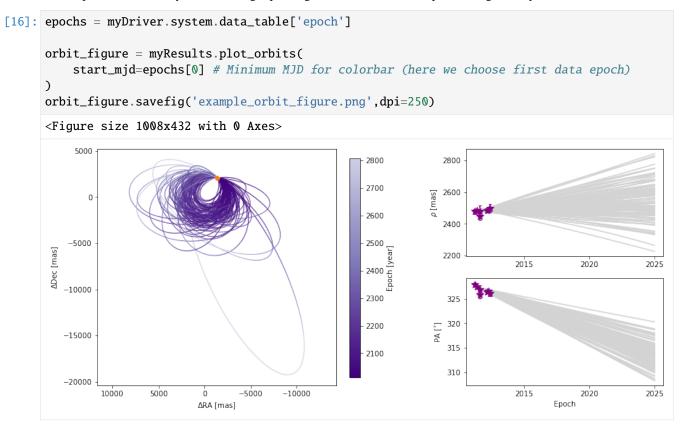


Overall, we find that some of the parameters have converged well but others are not well constrained by our fit. As mentioned above, the output of the plot\_corner() methods are matplotlib Figure objects. So, if we wanted to save this figure, we might use the savefig() method.

[15]: corner\_figure\_median\_95.savefig('plotting\_tutorial\_corner\_figure\_example.png')

#### 4.2 Orbit Plot

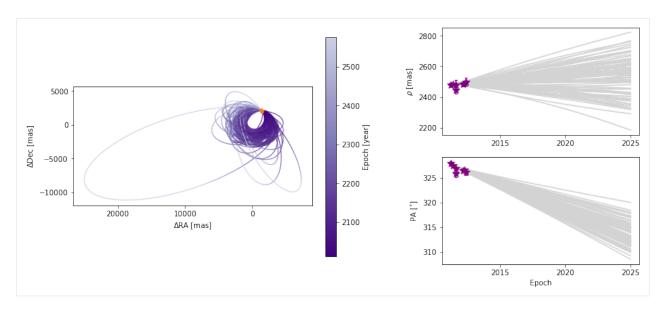
The plot\_orbits method in the Results module allows us to visualize the orbits sampled by orbitize!. The default call to plot\_orbits will draw 100 orbits randomly chosen out of the total orbits sampled (set by parameter num\_orbits\_to\_plot). In addition, to draw each of these orbits, by default, we will sample each orbit at 100 evenly spaced points in time throughout the orbit's orbital period (set by parameter num\_epochs\_to\_plot). These points are then connected by coloured line segments corresponding to the date where the object would be at that point in the orbit. By default, orbits are plotted starting in the year 2000 (set by parameter start\_mjd) and are drawn for one complete orbital period. We usually choose to begin plotting orbits at the first data epoch, using this keyword as illustrated below.



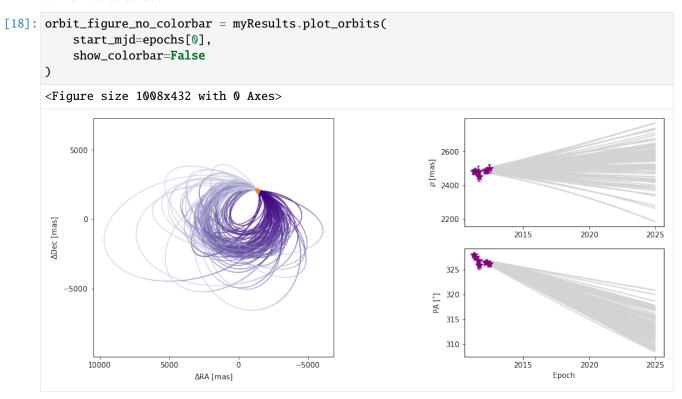
#### Customizing orbit plot appearence

In the above figure, we see that the x and y axes are RA and Dec offsets from the primary star in milliarcseconds. By default the axes aspect ratio is square, but we can turn that off. This is normally not recommended but for some situations, it may be desirable.

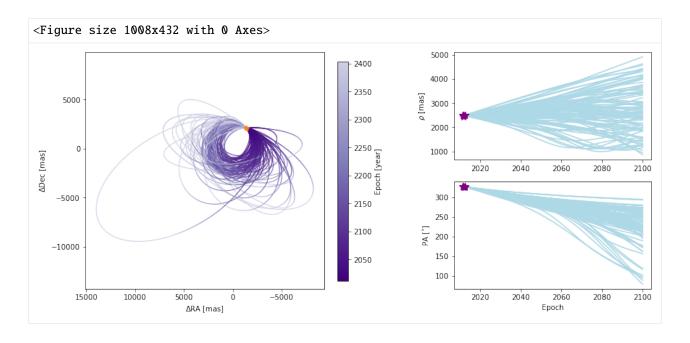
(Note: Each call to plot\_orbits selects a different random subset of orbits to plot, so the orbits shown in the figures here are not exactly the same each time).



The colorbar shows how the line segment colors correspond to the date, beginning at the first data epoch. We can also turn off the colourbar.

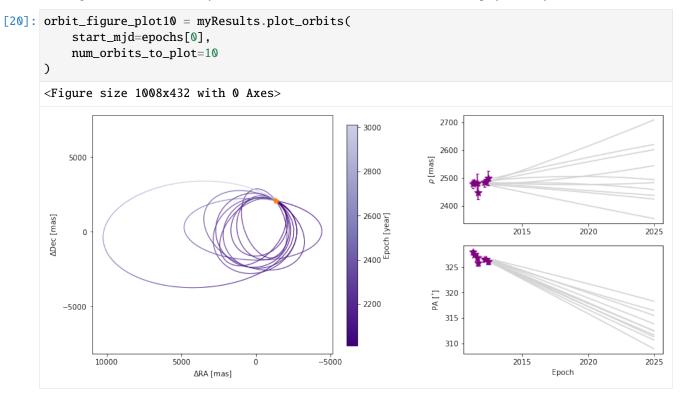


We can also modify the color and ending-epoch of the separation/position angle panels as follows:

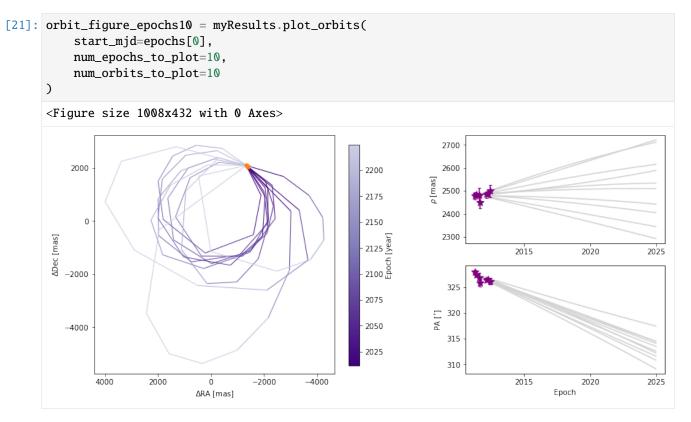


## Choosing how orbits are plotted

Plotting one hundred orbits may be too dense. We can set the number of orbits displayed to any other value.



We can also adjust how well we sample each of the orbits. By default, 100 evenly-spaced points are computed per orbit, beginning at start\_mjd and ending one orbital period later. Decreasing the sampling could lead to faster plot generation but if it is too low, it might not correctly sample the orbit, as shown below.



In this example, there is only one companion in orbit around the primary. When there are more than one, plot\_orbits will plot the orbits of the first companion by default and we would use the object\_to\_plot argument to choose a different object (where 1 is the first companion).

#### 4.3 Working with matplotlib Figure objects

The idea of the Results plotting functions is to create some basic plots and to return the matplotlib Figure object so that we can do whatever else we may want to customize the figure. We should consult the matplotlib API for all the details. Here, we will outline a few common tasks.

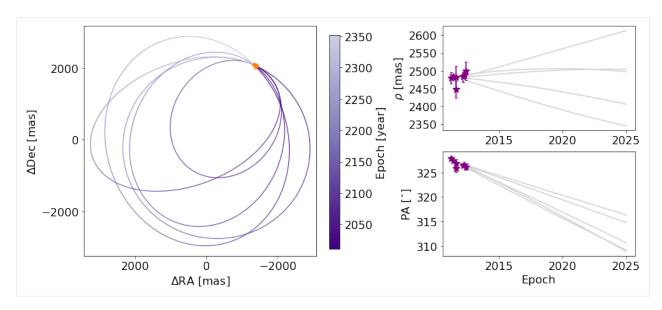
Let's increase the font sizes for all of the text (maybe for a poster or oral presentation) using matplotlib.pyplot. This (and other edits to the rcParams defaults) should be done before creating any figure objects.

```
[22]: plt.rcParams.update({'font.size': 16})
```

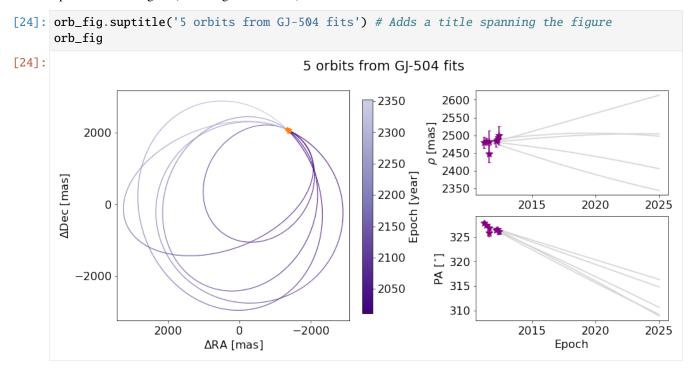
Now, we will start with creating a figure with only 5 orbits plotted, for simplicity, with the name orb\_fig. This Figure object has two axes, one for the orbit plot and one for the colorbar. We can use the .axes property to get a list of axes. Here, we've named the two axes ax\_orb and ax\_cbar. With these three objects (orb\_fig and ax\_orb, and ax\_cbar) we can modify all aspects of our figure.

```
[23]: orb_fig = myResults.plot_orbits(start_mjd=epochs[0], num_orbits_to_plot=5)
    ax_orb, ax_sep, ax_pa, ax_cbar = orb_fig.axes

<Figure size 1008x432 with 0 Axes>
```

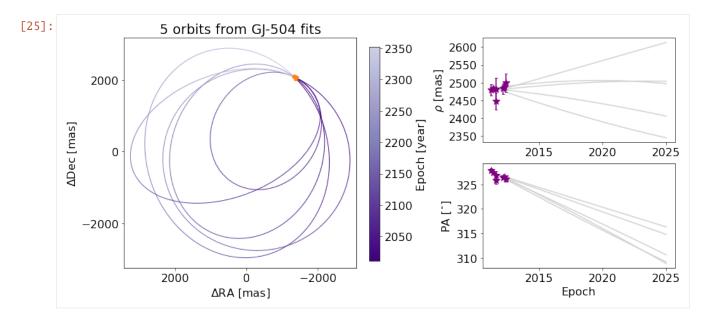


First, let's try to add a figure title. We have two options. We can use the Figure's suptitle method to add a title that spans the entire figure (including the colorbar).

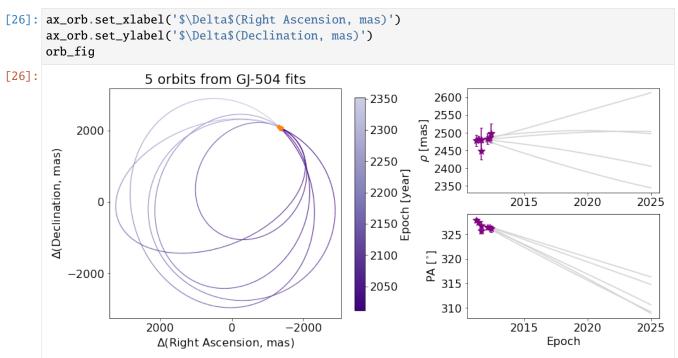


Alternatively, we can just add the title over the Ra/Dec axes instead.

[25]: orb\_fig.suptitle('') # Clears previous title
ax\_orb.set\_title('5 orbits from GJ-504 fits') # sets title over Axes object only
orb\_fig



We can also change the label of the axes, now using matplotlib.Axes methods.

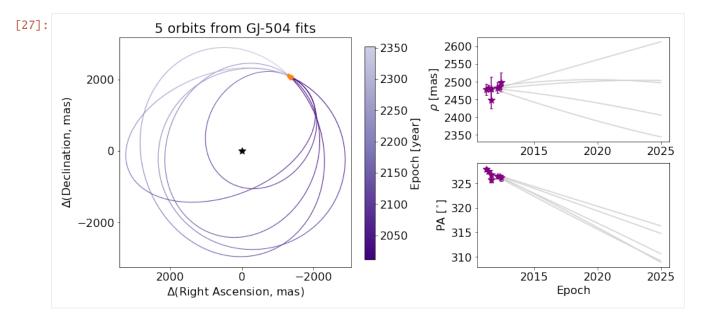


If we want to modify the colorbar axis, we need to access the ax\_cbar object

ax\_orb.set\_xlabel(' $\Delta RA$  [mas]') ax\_orb.set\_ylabel(' $\Delta Dec$  [mas]') # go back to what we had before ax\_cbar.set\_title('Year') # Put a title on the colorbar orb\_fig

We may want to add to the plot itself. Here's an exmaple of putting an star-shaped point at the location of our primary star.

```
[27]: ax_orb.plot(0,0,marker="*",color='black',markersize=10)
    orb_fig
```



And finally, we can save the figure objects.

[28]: orb\_fig.savefig('plotting\_tutorial\_plot\_orbit\_example.png')

# 2.2.7 MCMC vs OFTI Comparison

by Sarah Blunt, 2018

Welcome to the OFTI/MCMC comparison tutorial! This tutorial is meant to help you understand the differences between OFTI and MCMC algorithms so you know which one to pick for your data.

Before we start, I'll give you the short answer: **for orbit fractions less than 5%**, **OFTI is generally faster to converge than MCMC**. This is not a hard-and-fast statistical rule, but I've found it to be a useful guideline.

This tutorial is essentially an abstract of Blunt et al (2017). To dig deeper, I encourage you to read the paper (Sections 2.2-2.3 in particular).

**Goals of This Tutorial**: - Understand qualitatively why OFTI converges faster than MCMC for certain datasets. - Learn to make educated choices of backend algorithms for your own datasets.

**Prerequisites**: - This tutorial assumes knowledge of the orbitize API. Please go through at least the OFTI and MCMC introduction tutorials before this one. - This tutorial also assumes a qualitative understanding of OFTI and MCMC algorithms. I suggest you check out at least Section 2.1 of Blunt et al (2017) and this blog post before attempting to decode this tutorial.

**Jargon**: - I will often use **orbit fraction**, or the fraction of the orbit spanned by the astrometric observations, as a figure of merit. In general, OFTI will converge faster than MCMC for small orbit fractions. - **Convergence** is defined differently for OFTI and for MCMC (see the OFTI paper for details). An OFTI run needs to accept a statistically large number of orbits for convergence, since each accepted orbit is independent of all others. For MCMC, convergence is a bit more complicated. At a high level, an MCMC run has converged when all walkers have explored the entire parameter space. There are several metrics for estimating MCMC convergence (e.g. GR statistic, min Tz statistic), but we'll just estimate convergence qualitatively in this tutorial.

```
[1]: import numpy as np import matplotlib.pyplot as plt (continues on next page)
```

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```
import astropy.table
import time

np.random.seed(5)

from orbitize.kepler import calc_orbit
from orbitize import system, sampler
from orbitize.read_input import read_file
```

## **Generate Synthetic Data**

Let's start by defining a function to generate synthetic data. This will allow us to easily test convergence speeds for different orbit fractions. I'll include the number of observations and the noise magnitude as keywords; I encourage you to test out different values throughout the tutorial!

```
[2]: mtot = 1.2  # total system mass [M_sol]
    plx = 60.0 \# parallax [mas]
    def generate_synthetic_data(sma=30.0, num_obs=4, unc=10.0):
         """Generate an orbitize-table of synethic data
        Args:
             sma (float): semimajor axis (au)
             num_obs (int): number of observations to generate
             unc (float): uncertainty on all simulated RA & Dec measurements (mas)
        Returns:
             2-tuple:
                 - `astropy.table.Table`: data table of generated synthetic data
                 - float: the orbit fraction of the generated data
        # assumed ground truth for non-input orbital parameters
        ecc = 0.5 # eccentricity
        inc = np.pi / 4 # inclination [rad]
        argp = 0.0
        lan = 0.0
        tau = 0.8
        # calculate RA/Dec at three observation epochs
        observation_epochs = np.linspace(
             51550.0, 52650.0, num_obs
        # 'num_obs' epochs between ~2000 and ~2003 [MJD]
        num_obs = len(observation_epochs)
        ra, dec, _ = calc_orbit(
            observation_epochs, sma, ecc, inc, argp, lan, tau, plx, mtot
        )
        # add Gaussian noise to simulate measurement
        ra += np.random.normal(scale=unc, size=num_obs)
                                                                                 (continues on next page)
```

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```
dec += np.random.normal(scale=unc, size=num_obs)
# define observational uncertainties
ra_err = dec_err = np.ones(num_obs) * unc
# make a plot of the data
plt.figure()
plt.errorbar(ra, dec, xerr=ra_err, yerr=dec_err, linestyle="")
plt.xlabel("$\\Delta$ RA")
plt.ylabel("$\\Delta$ Dec")
# calculate the orbital fraction
period = np.sqrt((sma**3) / mtot)
orbit_coverage = (
    max(observation_epochs) - min(observation_epochs)
) / 365.25 # [yr]
orbit_fraction = 100 * orbit_coverage / period
data_table = astropy.table.Table(
    [observation_epochs, [1] * num_obs, ra, ra_err, dec, dec_err],
    names=("epoch", "object", "raoff", "raoff_err", "decoff", "decoff_err"),
)
# read into orbitize format
data_table = read_file(data_table)
return data_table, orbit_fraction
```

## **Short Orbit Fraction**

Let's use the function above to generate some synthetic data with a short orbit fraction, and fit it with OFTI:

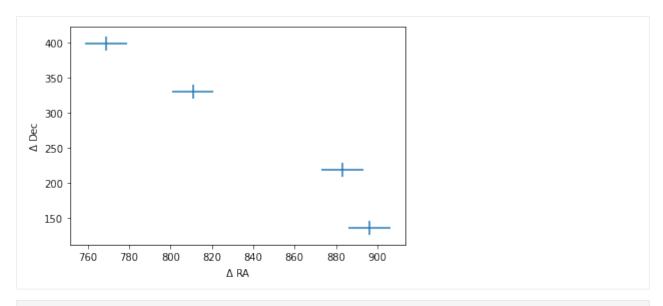
```
[3]: # generate data with default kwargs
    short_data_table, short_orbit_fraction = generate_synthetic_data()
    print("The orbit fraction is {}%".format(np.round(short_orbit_fraction), 1))

# initialize orbitize `System` object
    short_system = system.System(1, short_data_table, mtot, plx)

num2accept = 500 # run sampler until this many orbits are accepted

The orbit fraction is 2.0%
```

(continues on next page)



```
[4]: start_time = time.time()
    # set up OFTI `Sampler` object
    short_OFTI_sampler = sampler.OFTI(short_system)
    # perform OFTI fit
    short_OFTI_orbits = short_OFTI_sampler.run_sampler(num2accept)
    print(
         "OFTI took {} seconds to accept {} orbits.".format(
            time.time() - start_time, num2accept
        )
    Converting ra/dec data points in data_table to sep/pa. Original data are stored in input_
     ⊸table.
    497/500 orbits found
    KeyboardInterrupt
                                               Traceback (most recent call last)
    Input In [4], in <cell line: 7>()
          4 short_OFTI_sampler = sampler.OFTI(short_system)
          6 # perform OFTI fit
     ----> 7 short_OFTI_orbits = short_OFTI_sampler.run_sampler(num2accept)
          9 print(
         10
                 "OFTI took {} seconds to accept {} orbits.".format(
         11
                     time.time() - start_time, num2accept
         12
                 )
         13)
    File ~/Documents/GitHub/orbitize/orbitize/sampler.py:593, in OFTI.run_sampler(self,_
     →total_orbits, num_samples, num_cores, OFTI_warning)
        588
                        OFTI_warning = None
        589
                print(
         590
                    str(orbits_saved.value) + "/" + str(total_orbits) + " orbits found",
         591
                     end="\r",
```

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```
592 )
--> 593 time.sleep(0.1)
595 print(
596 str(total_orbits) + "/" + str(total_orbits) + " orbits found", end="\r"
597 )
599 # join the processes

KeyboardInterrupt:
```

```
[]: plt.hist(
         short_OFTI_orbits[:, short_system.param_idx["ecc1"]],
        bins=40,
        density=True,
        alpha=0.5,
        label="OFTI",
    plt.hist(
         short_MCMC_orbits[:, short_system.param_idx["ecc1"]],
        bins=40,
        density=True,
        alpha=0.5,
        label="MCMC",
    )
    plt.xlabel("Eccentricity")
    plt.ylabel("Prob.")
    plt.legend()
```

These distributions are different because the MCMC chains have not converged, resulting in a "lumpy" MCMC distribution. I set up the calculation so that MCMC would return 10x as many orbits as OFTI, but even so, the OFTI distribution is a much better representation of the underlying PDF.

If we run the MCMC algorithm for a greater number of steps (and/or increase the number of walkers and/or temperatures), the MCMC and OFTI distributions will become indistinguishable. **OFTI is NOT more correct than MCMC**, **but for this dataset, OFTI converges on the correct posterior faster than MCMC**.

## **Longer Orbit Fraction**

Let's now repeat this exercise with a longer orbit fraction. For this dataset, OFTI will have to run for several seconds just to accept one orbit, so we won't compare the resulting posteriors.

```
[]: # generate data
    long_data_table, long_orbit_fraction = generate_synthetic_data(sma=10, num_obs=5)
    print("The orbit fraction is {}%".format(np.round(long_orbit_fraction), 1))
    # initialize orbitize `System` object
    long_system = system.System(1, long_data_table, mtot, plx)
    num2accept = 500 # run sampler until this many orbits are accepted
[ ]: start_time = time.time()
    # set up OFTI `Sampler` object
    long_OFTI_sampler = sampler.OFTI(long_system)
    # perform OFTI fit
    long_OFTI_orbits = long_OFTI_sampler.run_sampler(1)
    print("OFTI took {} seconds to accept 1 orbit.".format(time.time() - start_time))
[]: start_time = time.time()
    # set up MCMC `Sampler` object
    num_walkers = 20
    long_MCMC_sampler = sampler.MCMC(long_system, num_temps=10, num_walkers=num_walkers)
    # perform MCMC fit
     _ = long_MCMC_sampler.run_sampler(num2accept, burn_steps=100)
    long_MCMC_orbits = long_MCMC_sampler.results.post
    print("MCMC took {} steps in {} seconds.".format(num2accept, time.time() - start_time))
[]: plt.hist(long_MCMC_orbits[:, short_system.param_idx["ecc1"]], bins=15, density=True)
    plt.xlabel("Eccentricity")
    plt.ylabel("Prob.")
```

It will take more steps for this MCMC to fully converge (see the MCMC tutorial for more detailed guidelines), but you can imagine that MCMC will converge much faster than OFTI for this dataset.

### **Closing Thoughts**

If you play around with the num\_obs, sma, and unc keywords in the generate\_synthetic\_data function and repeat this exercise, you will notice that the OFTI acceptance rate and MCMC convergence rate depend on many variables, not just orbit fraction. In truth, the Gaussianity of the posterior space determines how quickly an MCMC run will converge, and its similarity to the prior space determines how quickly an OFTI run will converge. In other words, the more constrained your posteriors are (relative to your priors), the quicker MCMC will converge, and the slower OFTI will run.

Orbit fraction is usually a great tracer of this "amount of constraint," but it's good to understand why!

**Summary**: - OFTI and MCMC produce the same posteriors, but often take differing amounts of time to converge on the correct solution. - OFTI is superior when your posteriors are similar to your priors, and MCMC is superior when your posteriors are highly constrained Gaussians.

# 2.2.8 Modifying MCMC Initial Positions

by Henry Ngo (2019) & Sarah Blunt (2021) & Mireya Arora (2021)

When you set up the MCMC Sampler, the initial position of your walkers are randomly determined. Specifically, they are uniformly distributed in your Prior phase space. This tutorial will show you how to change this default behaviour so that the walkers can begin at locations you specify. For instance, if you have an initial guess for the best fitting orbit and want to use MCMC to explore posterior space around this peak, you may want to start your walkers at positions centered around this peak and distributed according to an N-dimensional Gaussian distribution.

Note: This tutorial is meant to be read after reading the MCMC Introduction tutorial. If you are wondering what walkers are, you should start there!

The Driver class is the main way you might interact with orbitize! as it automatically reads your input, creates all the orbitize! objects needed to do your calculation, and defaults to some commonly used parameters or settings. However, sometimes you want to work directly with the underlying API to do more advanced tasks such as changing the MCMC walkers' initial positions, or modifying the priors.

This tutorial walks you through how to do that.

Goals of this tutorial: - Learn to modify the MCMC Sampler object - Learn about the structure of the orbitize code base

## Import modules

```
[1]: import numpy as np
from scipy.optimize import minimize as mn
import orbitize
from orbitize import driver
import multiprocessing as mp
```

### 1) Create Driver object

First, let's begin as usual and create our Driver object, as in the MCMC Introduction tutorial.

```
[2]: filename = "{}/GJ504.csv".format(orbitize.DATADIR)

# system parameters
num_secondary_bodies = 1
total_mass = 1.75 # [Msol]
plx = 51.44 # [mas]
mass_err = 0.05 # [Msol]
plx_err = 0.12 # [mas]

# MCMC parameters
num_temps = 5
num_walkers = 30
num_walkers = 30
num_threads = mp.cpu_count() # or a different number if you prefer
```

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```
my_driver = driver.Driver(
    filename,
    "MCMC",
    num_secondary_bodies,
    total_mass,
    plx,
    mass_err=mass_err,
    plx_err=plx_err,
    mcmc_kwargs={
        "num_temps": num_temps,
        "num_walkers": num_walkers,
        "num_threads": num_threads,
    },
)
```

#### 2) Access the Sampler object to view the walker positions

As mentioned in the introduction, the Driver class creates the objects needed for the orbit fit. At the time of this writing, it creates a Sampler object which you can access with the .sampler attribute and a System object which you can access with the .system attribute.

The Sampler object contains all of the information used by the orbit sampling algorithm (OFTI or MCMC) to fit the orbit and determine the posteriors. The System object contains information about the astrophysical system itself (stellar and companion parameters, the input data, etc.).

To see all of the attributes of the driver object, you can use dir().

```
[3]: print(dir(my_driver))

['__class__', '__delattr__', '__dict__', '__dir__', '__doc__', '__eq__', '__format__', '__

-__ge__', '__getattribute__', '__gt__', '__hash__', '__init__', '__init_subclass__', '__

--_-le__', '__lt__', '__module__', '__ne__', '__new__', '__reduce__', '__reduce_ex__', '__

----repr__', '__setattr__', '__sizeof__', '__str__', '__subclasshook__', '__weakref__',

-----sampler', 'system']
```

This returns many other functions too, but you see sampler and system at the bottom. Don't forget that in Jupyter notebooks, you can use my\_driver? to get the docstring for its class (i.e. the Driver class) and my\_driver?? to get the full source code of that class. You can also get this information in the API documentation.

Now, let's list the attributes of the my\_driver.sampler attribute.

Again, you can use the ? and ?? features as well as the API documentation to find out more. Here we see an attribute curr\_pos which contains the current position of all the walkers for the MCMC sampler. These positions were generated upon initialization of the Sampler object, which happened as part of the initialization of the Driver object.

#### Examine my\_driver.sampler.curr\_pos

curr\_pos is an array and has shape (n\_temps, n\_walkers, n\_params) for the parallel-tempered MCMC sampler and shape (n\_walkers, n\_params) for the affine-invariant ensemble sampler.

```
[5]: my_driver.sampler.curr_pos.shape # Here we are using the parallel-tempered MCMC sampler
[5]: (5, 30, 8)
```

Basically, this is the same shape as the output of the Sampler. Let's look at the start position of the first five walkers at the lowest temperature, to get a better sense of what the strucutre is like.

```
[6]: print(my_driver.sampler.curr_pos[0, 0:5, :])

[[6.95236402e+03 2.32633596e-01 1.60836175e+00 1.14721977e+00
5.34421956e+00 2.67696018e-01 5.13656024e+01 1.80562444e+00]
[7.98190639e-02 1.40596874e-01 2.63470503e+00 4.35734182e+00
2.28246569e-01 4.75070744e-01 5.13486020e+01 1.77552365e+00]
[1.60516586e-03 8.80915694e-01 2.28799219e+00 2.34160551e+00
3.54811275e+00 9.29570443e-01 5.14765240e+01 1.72740379e+00]
[1.19498384e+00 4.70033780e-01 1.15697304e+00 5.15452767e+00
2.26789404e+00 2.79424283e-01 5.13693594e+01 1.70312991e+00]
[1.63550027e-03 3.26684316e-01 1.71079639e+00 4.00183596e+00
1.81830795e+00 2.93151051e-01 5.16095100e+01 1.73093287e+00]]
```

## 3) Replace curr\_pos with your own initial positions for walkers

When the sampler is run with the sampler.run\_sampler() method, it will start the walkers at the curr\_pos values, run the MCMC forward for the given number of steps, and then update curr\_pos to reflect where the walkers ended up. The next time run\_sampler() is called, it does the same thing again.

Here, you have just created the sampler but have not run it yet. So, if you update curr\_pos with our own custom start locations, when you run the sampler, it will begin at your custom start locations instead.

#### 3.1) Generate your own initial positions

There are many ways to create your own walker start distribution and what you want to do will depend on your science question and prior knowledge.

If you have already generated and validated your own initial walker positions, you can skip down to the "Update sampler position". Some users use the output of OFTI or a previous MCMC run as the initial position.

If you need to generate your own positions, read on. Here, let's assume you know a possible best fit value and your uncertainty in that fit. Perhaps you got this through a least squares minimization. So, let's create a distribution of walkers that are centered on the best fit value and distributed normallly with the 1-sigma in each dimension equal to the uncertainty on that best fit value.

First, let's define the best fit value and the spread. As a reminder, the order of the parameters in the array is (for a single planet-star system): semimajor axis, eccentricity, inclination, argument of periastron, position angle of nodes, epoch of periastron passage, parallax and total mass. You can check the indices with this dict in the system object.

```
[7]: print(my_driver.system.param_idx)
{'sma1': 0, 'ecc1': 1, 'inc1': 2, 'aop1': 3, 'pan1': 4, 'tau1': 5, 'plx': 6, 'mtot': 7}
```

```
[8]: # Set centre and spread of the walker distribution
     # Values from Table 1 in Blunt et al. 2017, AJ, 153, 229
    sma_cen = 44.48
    sma_sig = 15.0
    ecc\_cen = 0.0151
    ecc\_sig = 0.175
    inc_cen = 2.30 # (131.7 deg)
    inc\_sig = 0.279 \# (16.0 deg)
    aop_cen = 1.60 # (91.7 deg)
    aop_sig = 1.05 # (60.0 deg)
    pan_cen = 2.33 \# (133.7 deg)
    pan\_sig = 0.872 \# (50.0 deg)
    tau_cen = 0.77 # (2228.11 yr)
    tau_sig = 0.65 \# (121.0 yr)
    # Note : parallax and stellar mass already defined above (plx, plx_err, total_mass, mass_
     \hookrightarrowerr)
    walker_centres = np.array(
         [sma_cen, ecc_cen, inc_cen, aop_cen, pan_cen, tau_cen, plx, total_mass]
    walker_1sigmas = np.array(
         [sma_sig, ecc_sig, inc_sig, aop_sig, pan_sig, tau_sig, plx_err, mass_err]
    )
```

You can use numpy.random.standard\_normal to generate normally distributed random numbers in the same shape as your walker initial positions (my\_driver.sampler.curr\_pos.shape). Then, multiply by walker\_1sigmas to get the spread to match your desired distribution and add walker\_centres to get the distribution centered on your desired values.

```
[9]: curr_pos_shape = my_driver.sampler.curr_pos.shape # Get shape of walker positions
# Draw from multi-variate normal distribution to generate new walker positions
new_pos = np.random.standard_normal(curr_pos_shape) * walker_1sigmas + walker_centres
```

## 3.2) Using an optimizer to obtain a best fit value

Other optimizing software can also be used to generate intial positions. Depending on the quality of data collected and whether a suitable guess array of parameters can be made, different optimizing software can provide better best fit values for for MCMC walkers. Below you will find a few options that cater to different scenarios.

### 3.2a) Using scipy.optimize.minimize

Assuming the data obtained allows for a suitable guess to be made for each parameter, a scipy.optimize.minimize software can be used to generate a best fit value. You may want to skip this step and input your guess values directly into MCMC's initial walker positions, however scipy can help refine the guess parameters.

First, we define a new log liklihood function function neg\_logl based on the guess values we have. Note, since we have predefined a good guess, from the aforementioned Table, as walker\_centres we will continue to use it as a guess array for examples below.

```
[10]: # The following code performs a minimization whereas the log liklihood function is based.
      →on maximization so we redefine the
      # likelihood function is redefined to return -x to make this a minization scenario
     m = my_driver.sampler
     def neg_logl(paramarray):
         x = m.logl(
             paramarray, include_logp=True
         ) # set include_logp to true to include guess array in likelihood function
         return -x
     guessarray = walker_centres
     results = mn(neg_log1, guessarray, method="Powel1")
     print(results.x) # results.x is the best fit value
      /home/docs/checkouts/readthedocs.org/user_builds/orbitize/envs/orbitize-manual/lib/
      →python3.10/site-packages/scipy/optimize/_optimize.py:2577: RuntimeWarning: invalid_
      →value encountered in scalar multiply
       tmp2 = (x - v) * (fx - fw)
      /home/docs/checkouts/readthedocs.org/user_builds/orbitize/envs/orbitize-manual/lib/
      →python3.10/site-packages/orbitize/priors.py:463: RuntimeWarning: invalid value
      →encountered in log
       lnprob = np.log(np.sin(element_array)/normalization)
      [4.90426906e+01 9.99444988e-06 2.48501010e+00 1.60085022e+00
      2.33034111e+00 7.69934177e-01 5.14404237e+01 1.74922411e+00]
```

In our trials, Powell has given the best results, but you may replace it with a different minimizing method depending on your need.

#### 3.3) Scattering walkers

To set up MCMC so that it explores the nearby probablity space thoroughly and finds the global minimum, you can scatter the initial positions of the walkers around the best fit value. This can be done by adding random numbers to results.x

This section overrides walker\_1 sigmas and creates a spread of new\_pos in a different manner than above. The following is a template based on the aforementioned Table. The scatter is created using a variety of methods, we recommend reviewing the code to ensure it is compatible to your data.

```
[11]: new_pos = np.random.standard_normal(curr_pos_shape) * 0.03 + results.x
```

### 3.4) Update sampler position

After generating and validating your new walker positions, through whatever methods you choose, it's now time to update the sampler object to have its curr\_pos be your new positions.

```
[12]: my_driver.sampler.curr_pos = np.copy(new_pos)
```

### 3.5) Validate your new positions

Drawing from a normal distribution can cause your walkers to start outside of your prior space. See the Modifying Priors tutorial for information on how to interact with the prior objects, which would allow you to find the limits on each parameter set by the priors etc.

Here, let's do something more simple and just check that all values are physically valid. After this we can begin to correct them.

The following function can be used to identify walkers that have been initialized outside of the appropriate prior probability space. It will raise a ValueError if walkers are initialized outside of the priors. You should update your positions until this method runs without raising an error.

```
[13]: try:
    my_driver.sampler.check_prior_support()
    except Exception as e:
        print(e)

Attempting to start with walkers outside of prior support: check parameter(s) 1
```

We should continue investigating which parameters are being initialized outside of the prior space until this function returns empty lists.

And you're done! You can continue at "Running the MCMC Sampler" in the MCMC Introduction Tutorial

# 2.2.9 Radial Velocity Tutorial for MCMC

By Roberto Tejada (2019)

This tutorial will assume the user is familiar with the Driver class and is acquainted with MCMC terminology. For more information about MCMC, see the MCMC Introduction Tutorial.

We explain how to jointly fit radial velocity data and relative astrometry using the MCMC technique. First we need a set of data containing radial velocity measurements. We check the data using read\_input and observe the quant\_type column for radial velocity data. For more information on orbitize.read\_input.read\_file(), see the Formatting Input Tutorial. You can fit for separate jitter and gamma terms for each RV instrument in your dataset by adding an "instrument" column to your data csv.

**NOTE**: Astrometry+RV fitting currently only works with MCMC and not OFTI.

## **Read and Format Data**

```
[1]: import numpy as np
import matplotlib.pyplot as plt
from orbitize import read_input, plot, priors, driver, DATADIR
import multiprocessing as mp

data_table = read_input.read_file("{}/HD4747.csv".format(DATADIR)) # print all columns
data_table.pprint_all()
```

epoch	object	quant1	quant1_err	quant2	quant2_err	quant12_corr	quant_type	instrument
56942.3	1	606.5	7.0	180.04	0.62	nan	seppa	defsp
57031.2	1	606.6	6.4	180.52	0.58	nan	seppa	defsp
57289.4	1	604.0	7.0	184.9	0.9	nan	seppa	defsp
50366.475	0	-0.54103	0.00123	nan	nan	nan	rv	defrv
50418.329	0	-0.40053	0.00206	nan	nan	nan	rv	defrv
50462.252	0	-0.24094	0.0011	nan	nan	nan	rv	defrv
50689.604	0	0.37292	0.00117	nan	nan	nan	rv	defrv
50784.271	0	0.46223	0.00133	nan	nan	nan	rv	defrv
50806.227	0	0.48519	0.00103	nan	nan	nan	rv	defrv
50837.217	0	0.49395	0.00117	nan	nan	nan	rv	defrv
50838.201	0	0.49751	0.00112	nan	nan	nan	rv	defrv
50839.211	0	0.50187	0.00112	nan	nan	nan	rv	defrv
51009.62	0	0.53355	0.00135	nan	nan	nan	rv	defrv
51011.548	0	0.53164	0.00128	nan	nan	nan	rv	defrv
51013.602	0	0.53629	0.0016	nan	nan	nan	rv	defrv
51050.491	0	0.52154	0.00468	nan	nan	nan	rv	defrv
51170.248	0	0.50757	0.0014	nan	nan	nan	rv	defrv
51367.585	0	0.47678	0.00131	nan	nan	nan	rv	defrv
51409.527	0	0.46147	0.00523	nan	nan	nan	rv	defrv
51543.244	0	0.44311	0.00152	nan	nan	nan	rv	defrv
51550.229	0	0.43286	0.00147	nan	nan	nan	rv	defrv
51755.551	0	0.39329	0.00213	nan	nan	nan	rv	defrv
51899.284	0	0.36457	0.00163	nan	nan	nan	rv	defrv
52097.626	0	0.32986	0.00157	nan	nan	nan	rv	defrv
52488.542	0	0.26687	0.0015	nan	nan	nan	rv	defrv
52572.312	0	0.25035	0.00195	nan	nan	nan	rv	defrv
52987.228	0	0.19466	0.00238	nan	nan	nan	rv	defrv
52988.186	0	0.18469	0.00194	nan	nan	nan	rv	defrv
53238.456	0	0.16892	0.00134	nan	nan	nan	rv	defrv
53303.403	0	0.16769	0.00112	nan	nan	nan	rv	defrv
53339.265	0	0.16069	0.00119	nan	nan	nan	rv	defrv
53724.274	0	0.11302	0.00103	nan	nan	nan	rv	defrv
53724.276	0	0.11605	0.00112	nan	nan	nan	rv	defrv
54717.455	0	0.00984	0.00123	nan	nan	nan	rv	defrv
54718.508	0	0.01242	0.00115	nan	nan	nan	rv	defrv
54719.51	0	0.01572	0.00123	nan	nan	nan	rv	defrv
54720.47	0	0.01534	0.00113	nan	nan	nan	rv	defrv
54722.401	0	0.01479	0.00127	nan	nan	nan	rv	defrv
54723.47	0	0.01422	0.00122	nan	nan	nan	rv	defrv
54724.474	0	0.01169	0.0012	nan	nan	nan	rv	defrv
54725.383	0	0.01383	0.00113	nan	nan	nan	rv	defrv

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(confini	ied from	previous	nage)

54726.505	0 0.0195	0.00123	nan	nan	nan	rv	defrv
54727.452	0 0.0175	0.00113	nan	nan	nan	rv	defrv
55014.62	0 -0.00636	0.00141	nan	nan	nan	rv	defrv
55015.624	0 -0.00409	0.00138	nan	nan	nan	rv	defrv
55016.624	0 -0.00566	0.00121	nan	nan	nan	rv	defrv
55048.525	0 -0.01975	0.00124	nan	nan	nan	rv	defrv
55076.584	0 -0.01614	0.00128	nan	nan	nan	rv	defrv
55077.594	0 -0.01303	0.00126	nan	nan	nan	rv	defrv
55134.464	0 -0.01689	0.00136	nan	nan	nan	rv	defrv
55198.254	0 -0.02885	0.0012	nan	nan	nan	rv	defrv
55425.584	0 -0.04359	0.00125	nan	nan	nan	rv	defrv
55522.379	0 -0.0512	0.0013	nan	nan	nan	rv	defrv
55806.547	0 -0.07697	0.0013	nan	nan	nan	rv	defrv
56148.55	0 -0.10429	0.00128	nan	nan	nan	rv	defrv
56319.201	0 -0.1102	0.00128	nan	nan	nan	rv	defrv
56327.208	0 -0.11332	0.00149	nan	nan	nan	rv	defrv
56507.645	0 -0.12324	0.00133	nan	nan	nan	rv	defrv
56912.534	0 -0.17085	0.00113	nan	nan	nan	rv	defrv

The quant\_type column displays the type of data each row contains: astrometry (radec or seppa), or radial velocity (rv). For astrometry, quant1 column contains right ascension or separation, and the quant2 column contains declination or position angle. For rv data, quant1 contains radial velocity data in  $\rm km/s$ , while quant2 is filled with nan to preserve the data structure. The table contains each respective error column.

We can now initialize the Driver class. MCMC samplers take time to converge to absolute maxima in parameter space, and the more parameters we introduce, the longer we expect it to take.

## **Create Driver Object**

For joint orbital fits with RV data, we need to fit the stellar and companion masses (m0 and m1 respectively as separate free parameters). This differs from the astrometry-only case where fitting the total mass mtot suffices. We set the system keyword fit\_secondary\_mass to True when initializing the Driver object.

```
[2]: filename = "{}/HD4747.csv".format(DATADIR)
    # system parameters
    num_secondary_bodies = 1
    stellar_mass = 0.84 # [Msol]
    plx = 53.18 \# [mas]
    mass\_err = 0.04 \# [Msol]
    plx_err = 0.12 # [mas]
    # MCMC parameters
    num\_temps = 5
    num_walkers = 30
    num_threads = 2 # or a different number if you prefer, eg mp.cpu_count()
    my_driver = driver.Driver(
         filename, 'MCMC', num_secondary_bodies, stellar_mass, plx, mass_err=mass_err, plx_
     →err=plx_err,
         system_kwargs = {'fit_secondary_mass':True, 'tau_ref_epoch':0},
        mcmc_kwargs={'num_temps': num_temps, 'num_walkers': num_walkers, 'num_threads': num_
                                                                                  (continues on next page)
```

(continued from previous page)

```
→threads}
```

Since MCMC is an object in orbitize!, we can assign a variable to the sampler and work with this variable:

```
[3]: m = my_driver.sampler
```

#### **RV Priors**

The priors for the two RV parameters, the radial velocity offset (gamma), and jitter (sigma), have default uniform prior and log uniform prior respectively. The gamma uniform prior is set between (-5,5) km/s, and the jitter log uniform prior is set for  $(10^{-4},0.05)$  km/s. The prior for m1 is a log uniform prior and is set for  $(10^{-3},2.0)M_{\odot}$ . The current version of orbitize addressed in this tutorial returns the stellar radial velocity only.

**NOTE**: We may change the priors as instructed in the Modifying Priors tutorial:

### **Running the MCMC Sampler**

As noted in the MCMC Introduction Tutorial, we must choose the sampler step for MCMC and can save every nthsample to avoid using too much disk space using thin.

```
[6]: total_orbits = 1000 # number of steps x number of walkers (at lowest temperature)
    burn_steps = 10 # steps to burn in per walker
    thin = 2 # only save every 2 steps
```

[7]: m.run\_sampler(total\_orbits, burn\_steps=burn\_steps, thin=thin) Starting Burn in /home/sblunt/Projects/orbitize/orbitize/priors.py:354: RuntimeWarning: invalid value →encountered in log lnprob = -np.log((element\_array\*normalizer)) /home/sblunt/Projects/orbitize/orbitize/priors.py:354: RuntimeWarning: invalid value\_ →encountered in log lnprob = -np.log((element\_array\*normalizer)) /home/sblunt/Projects/orbitize/orbitize/priors.py:463: RuntimeWarning: invalid value\_ →encountered in log lnprob = np.log(np.sin(element\_array)/normalization) /home/sblunt/Projects/orbitize/orbitize/priors.py:463: RuntimeWarning: invalid value\_ ⇔encountered in log lnprob = np.log(np.sin(element\_array)/normalization) 10/10 steps of burn-in complete Burn in complete. Sampling posterior now.

Run complete

Now we can plot the distribution of MCMC parameter of interest:

```
[8]: accepted_m1 = m.results.post[:, lab['m1']]
     plt.hist(accepted_m1,histtype='step')
     plt.xlabel('m1'); plt.ylabel('number of orbits')
     plt.show()
         400
      number of orbits
         300
         200
         100
           0
               0.0
                       0.2
                                0.4
                                         0.6
                                                  0.8
                                                           1.0
                                                                   1.2
                                        m1
```

## Saving Results over Extended MCMC Run

Sometimes our MCMC run will need to run for an extended period of time to let the walkers converge. To observe the convergence, we often need to see the walkers' progress along parameter space. We can save the sampler results periodically and keep running the sampler until convergence. To run for a greater number of steps and periodically save the results, we can create a for-loop and run for as many iterations as we'd like.

```
[9]: filename = "{}/HD4747.csv".format(DATADIR)
    total_orbits = 1000 # number of steps x number of walkers (at lowest temperature)
    burn_steps = 10 # steps to burn in per walker
    thin = 2 # only save every 10th step
    # system parameters
    num_secondary_bodies = 1
    stellar_mass = 0.84 # [Msol]
    plx = 53.18 \# [mas]
    mass\_err = 0.04 \# [Msol]
    plx_err = 0.12 # [mas]
    # MCMC parameters
    num\_temps = 5
    num_walkers = 30
    num_threads = 2 # or a different number if you prefer, e.g. mp.cpu_count()
    my_driver = driver.Driver(
        filename, 'MCMC', num_secondary_bodies, stellar_mass, plx, mass_err=mass_err, plx_
     →err=plx_err,
        system_kwargs = {'fit_secondary_mass':True, 'tau_ref_epoch':0},
        mcmc_kwargs={'num_temps': num_temps, 'num_walkers': num_walkers, 'num_threads': num_
     →threads}
    m = my_driver.sampler
```

We're now ready for the loop! The results object contains a save\_results function which lets us save the results for our directory, and we will use the load\_results object from results to access the data later. We also define the n\_iter below to mark how many MCMC runs to save our within results.

**NOTE**: To avoid long convergence periods, we may initialize the walkers in a sphere around the global minima of the parameter space as outlined in our Modifying MCMC Initial Positions Tutorial.

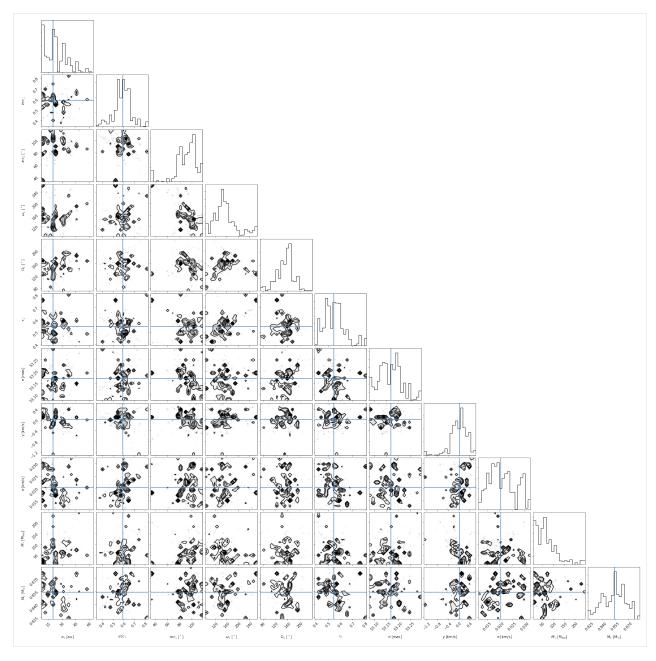
```
[10]: # file name to save as:
hdf5_filename = 'my_rv_posterior_%1d.hdf5'

[11]: n_iter = 2 # number of iterations
for i in range(n_iter):
    # running the sampler:
    orbits = m.run_sampler(total_orbits, burn_steps=burn_steps, thin=thin)
    myResults = m.results
    hdf5_filename = 'my_rv_posterior_%1d.hdf5' % i
    myResults.save_results(hdf5_filename) # saves results object as an hdf5 file
```

```
Starting Burn in
/home/sblunt/Projects/orbitize/orbitize/priors.py:354: RuntimeWarning: invalid value
→encountered in log
  lnprob = -np.log((element_array*normalizer))
/home/sblunt/Projects/orbitize/orbitize/priors.py:463: RuntimeWarning: invalid value_
→encountered in log
  lnprob = np.log(np.sin(element_array)/normalization)
/home/sblunt/Projects/orbitize/orbitize/priors.py:354: RuntimeWarning: invalid value_
→encountered in log
  lnprob = -np.log((element_array*normalizer))
/home/sblunt/Projects/orbitize/orbitize/priors.py:463: RuntimeWarning: invalid value
→encountered in log
 lnprob = np.log(np.sin(element_array)/normalization)
10/10 steps of burn-in complete
Burn in complete. Sampling posterior now.
Run complete
Starting Burn in
/home/sblunt/Projects/orbitize/orbitize/priors.py:354: RuntimeWarning: invalid value.
→encountered in log
  lnprob = -np.log((element_array*normalizer))
/home/sblunt/Projects/orbitize/orbitize/priors.py:354: RuntimeWarning: invalid value_
→encountered in log
  lnprob = -np.log((element_array*normalizer))
/home/sblunt/Projects/orbitize/orbitize/priors.py:463: RuntimeWarning: invalid value_
→encountered in log
  lnprob = np.log(np.sin(element_array)/normalization)
/home/sblunt/Projects/orbitize/orbitize/priors.py:463: RuntimeWarning: invalid value_
→encountered in log
  lnprob = np.log(np.sin(element_array)/normalization)
10/10 steps of burn-in complete
Burn in complete. Sampling posterior now.
Run complete
```

### **Plotting and Accesing Saved Results**

We can plot the corner plot saved in the results object by following the steps in the Advanced Plotting Tutorial:



As illustrated in the plot above, MCMC needs more time to run. We only performed two iterations in the loop to demonstrate its useage, but with increased n\_iter, the trendplots saved in the loop and the corner plot will show how the walkers converge to absolute extrema in parameter space.

To access the saved data, we can read it into a results object as shown in the MCMC Introduction Tutorial:

```
[13]: from orbitize import results
loaded_results = results.Results() # Create blank results object for loading
loaded_results.load_results('my_rv_posterior_%1d.hdf5' % (n_iter-1))
```

To demonstrate use of the loaded results file above, we can use the saved results to plot our orbital plots:

```
[14]: epochs = my_driver.system.data_table['epoch']
      orbit_plot_fig = loaded_results.plot_orbits(
          object_to_plot = 1, # Plot orbits for the first (and only, in this case) companion
          num_orbits_to_plot= 50, # Will plot 50 randomly selected orbits of this companion
          start_mjd=epochs[0], # Minimum MJD for colorbar (here we choose first data epoch)
      )
      WARNING: ErfaWarning: ERFA function "d2dtf" yielded 1 of "dubious year (Note 5)".
      →[astropy._erfa.core]
      WARNING: ErfaWarning: ERFA function "dtf2d" yielded 1 of "dubious year (Note 6)".
      →[astropy._erfa.core]
      WARNING: ErfaWarning: ERFA function "utctai" yielded 1 of "dubious year (Note 3)"
      →[astropy._erfa.core]
      WARNING: ErfaWarning: ERFA function "taiutc" yielded 1 of "dubious year (Note 4)"
      →[astropy._erfa.core]
      WARNING: ErfaWarning: ERFA function "dtf2d" yielded 10 of "dubious year (Note 6)".
      →[astropy._erfa.core]
      <Figure size 1008x432 with 0 Axes>
                                                                    1500
          4000
                                                        2160
                                                                   1000
                                                       2140
          2000
                                                                    500
                                                        2120
                                                                      0
       Mec [mas
                                                        2100 🕺
                                                                           2016
                                                                                2018
                                                                                      2020
                                                                                           2022
                                                                                                 2024
            0
                                                       2080 🖺
                                                                    300
         -2000
                                                       2060
                                                                  200
                                                       2040
                                                                    100
         -4000
                                                        2020
                                                                      0
               4000
                      2000
                                    -2000
                                           -4000
                                                                           2016
                                                                                2018
                                                                                      2020
                                                                                           2022
                                                                                                 2024
                             ΔRA [mas]
```

We can pass the rv\_time\_series = True argument in plot\_orbits to display the RV time series plot as a third panel of plot\_orbits:

```
WARNING: ErfaWarning: ERFA function "d2dtf" yielded 1 of "dubious year (Note 5)"
→[astropy._erfa.core]
WARNING: ErfaWarning: ERFA function "dtf2d" yielded 1 of "dubious year (Note 6)"
→[astropy._erfa.core]
WARNING: ErfaWarning: ERFA function "utctai" yielded 1 of "dubious year (Note 3)"
→[astropy._erfa.core]
WARNING: ErfaWarning: ERFA function "taiutc" yielded 1 of "dubious year (Note 4)"
→[astropy._erfa.core]
WARNING: ErfaWarning: ERFA function "dtf2d" yielded 4 of "dubious year (Note 6)"
→[astropy._erfa.core]
<Figure size 1008x432 with 0 Axes>
                                                  2050
                                                         1500
    2000
                                                  2040
                                                        1000
                                                         500
      0
                                                  2030
ADec [mas]
                                                                             2010
                                                                                          2020
                                                                 2000
                                                  2020 음
   -2000
                                                         300
                                                  2010
                                                         200
   -4000
                                                         100
                                                  2000
         4000
                  2000
                            Ò
                                   -2000
                                            -4000
                                                                 2000
                                                                             2010
                                                                                          2020
                                                                              Epoch
                        ΔRA [mas]
    0.5
```

# 2.2.10 Multi-planet Fits

2000

2005

by Jason Wang (2020)

RV [km/s]

In orbitize!, we can fit for multiple planets in the system in the same way as fitting a single planet. Note that currently, orbitize! handles planet-planet gravitational interations with an nbody integrator solver. However, here, we only take into account star-planet interactions. For example, the distance between planet b and the star will change with the existance of planet c because planet c has some finite mass and perturbs the star from the system barycenter. Even without planet-planet scattering, one can approximately fit for dynamical masses this way. By default in orbitize!, we assume the planets are test particles (mass = 0), so there is no star-planet interactions. Later in this tutorial in the "Multiplanet Dynamical Mass" section, we will describe how to turn on this feature.

2010

Epoch

2015

Multi-planet capabilities are generally handled under the hood, requiring not many modifications to the procedure for fitting a single planet. In this example, we will fit a few measurements of the HR 8799 b and c planets.

2025

```
[1]: import os import orbitize import orbitize.driver
```

In this tutorial, we will do an example with OFTI just because it is fast. However, in most cases, you will likely want to use MCMC as OFTI slows down significantly with multiple planets (even if each planet only has two astrometric data points as shown in the example). MCMC also requires longer run time typically, but it generally scales better than OFTI.

We follow the same steps as in the OFTI tutorial but set number of secondary bodies to 2 and read in a data file that contains astrometry for two planets in the system (in the example, a shortened version of the HR 8799 b and c astrometry). For MCMC, do the same thing as the single planet MCMC tutorial and make the same challenges as we have here with OFTI. In summary, all that needs to be done is to include both planets' measurements in the input data file and adjust the number of secondary bodies in the system.

```
[2]: input_file = os.path.join(orbitize.DATADIR, "test_val_multi.csv")

my_driver = orbitize.driver.Driver(input_file, 'OFTI',

2, # number of secondary bodies in system

1.52, # total mass [M_sun]

24.76, # total parallax of system [mas]

mass_err=0.15,

plx_err=0.64)

Converting ra/dec data points in data_table to sep/pa. Original data are stored in input_

→ table.
```

Next we run the sampler as usual:

```
[3]: s = my_driver.sampler
orbits = s.run_sampler(1000)
```

With two planets, we have 2 sets of 6 orbital parameters as well as the 2 system parameters (parallax and total mass). Our posterior, stored in orbits, is a (1000 x 14) array instead of the (1000 x 8) array had we fit a single planet.

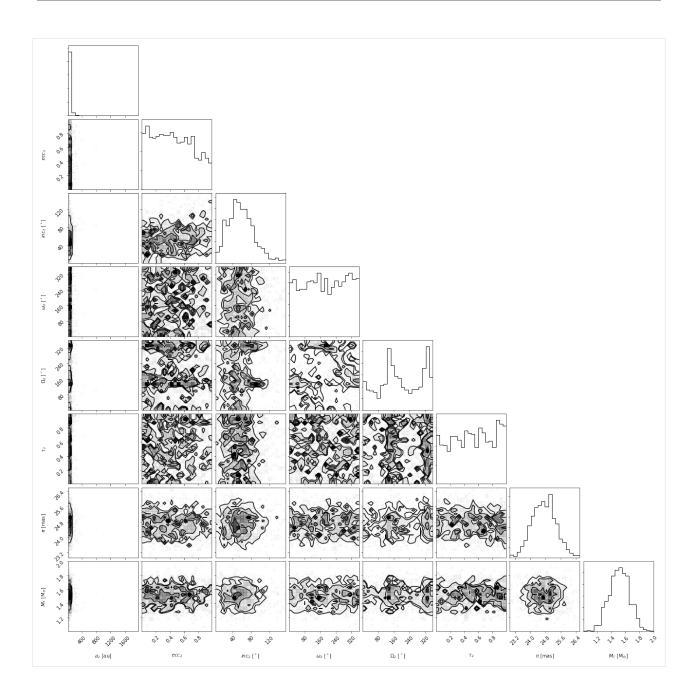
As it gets confusing to track with index corresponds to which orbital parameter, we recommend you use the system.param\_idx to index the parameters you are interested in. As a reminder, the abbreviations are: semi-major axis (sma), eccentricity (ecc), inclination (inc), argument of periastron (aop), position angle of nodes (pan), and epoch of periastron passage in fraction of the orbital period (tau). The 1 and 2 correspond to the two secondary bodies (in this case, HR 8799 b and HR 8799 c respectively)

## **Plotting**

We will go over briefly some considerations for visualizing multiplanet fits using the orbitize! API. For a more detailed guide on data visualization capabilities within orbitize, see the Orbitize plotting tutorial.

## **Corner Plot**

Corner plots are slow when trying to plot too many parameters (the number of subplots scales as n^2 where n is the number of dimensions). It also is difficult to read a plot with too many subplots. For this reason, we recommend looking at particular parameter covariances, or breaking it up to have one corner plot for each planet. Again, use the notation in system.param\_idx notation to grab the parameters you want.



# **Orbit Plot**

Currently, the orbit plotting tool in results class only plots the orbit of one body at a time. You can select which body you wish to plot.

```
[6]: epochs = my_driver.system.data_table['epoch']

orbit_figure = my_results.plot_orbits(
    start_mjd=epochs[0], # Minimum MJD for colorbar (here we choose first data epoch),
    object_to_plot=2 # plot planet c
)
```

```
WARNING: ErfaWarning: ERFA function "d2dtf" yielded 1 of "dubious year (Note 5)"

[astropy._erfa.core]
WARNING: ErfaWarning: ERFA function "dtf2d" yielded 1 of "dubious year (Note 6)"

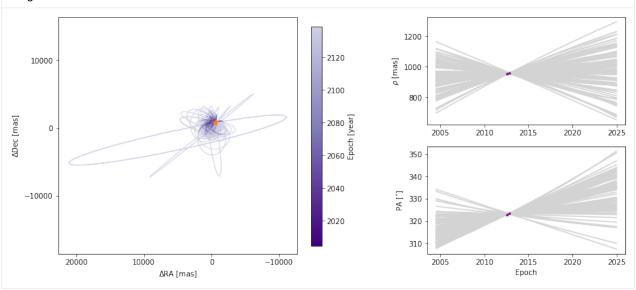
[astropy._erfa.core]
WARNING: ErfaWarning: ERFA function "utctai" yielded 1 of "dubious year (Note 3)"

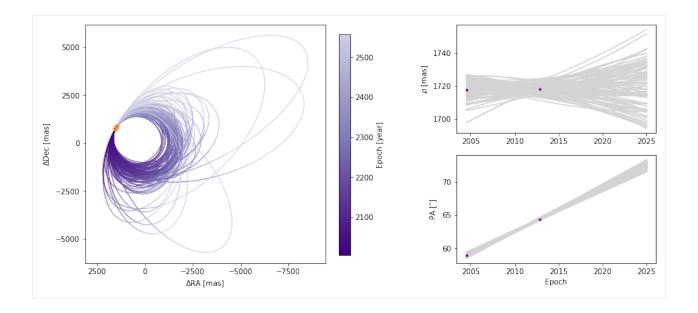
[astropy._erfa.core]
WARNING: ErfaWarning: ERFA function "taiutc" yielded 1 of "dubious year (Note 4)"

[astropy._erfa.core]
WARNING: ErfaWarning: ERFA function "dtf2d" yielded 5 of "dubious year (Note 6)"

[astropy._erfa.core]
```

#### <Figure size 1008x432 with 0 Axes>





# **Multiplanet Dynamical Mass**

In the case we want to fit dynamical masses, the procedure again remains unchanged as for a single planet system (as described in the RV MCMC Tutorial). The only thing to be aware of the extra parameters for the individual masses of the components. Again, use the system.param\_idx dictionary to keep track of the indices instead of trying to keep track of the ordering in one's head. Here, we will not demo a fit, but just show the increasing of parameters by 2 when fitting for the masses of the two secondary bodies.

Here, the new parameters are m1 and m2, the masses of planets b and c. m0 remains the mass of the star.

# 2.2.11 Using non-orbitize! Posteriors as Priors

By Jorge Llop-Sayson (2021)

This tutorial shows how to use posterior distribution from any source as orbitize! priors using a Kernel Density Estimator (KDE).

The user will need their posterior chains, consisiting of any number of correlated parameters, which will be used to get a KDE fit of the chains to be used as priors to orbitize!.

Once the priors are initialized the user can select their favorite fit method.

#### **Read Data**

```
[1]: import numpy as np
    import matplotlib.pyplot as plt
    from orbitize import system, priors, basis, read_input, DATADIR
    import pandas as pd
    # Read RadVel posterior chain
    pdf_fromRadVel = pd.read_csv(
         "{}/sample_radvel_chains.csv.bz2".format(DATADIR), compression="bz2", index_col=0
    per1 = pdf_fromRadVel.per1 # Period
    k1 = pdf_fromRadVel.k1 # Doppler semi-amplitude
    secosw1 = pdf_fromRadVel.secosw1
    sesinw1 = pdf_fromRadVel.sesinw1
    tc1 = pdf_fromRadVel.tc1 # time of conj.
    len_pdf = len(pdf_fromRadVel)
    /Users/bluez3303/miniconda3/envs/python3.10/lib/python3.10/site-packages/scipy/__init__.
     →py:146: UserWarning: A NumPy version >=1.17.3 and <1.25.0 is required for this version.
     →of SciPy (detected version 1.25.0
      warnings.warn(f"A NumPy version >={np_minversion} and <{np_maxversion}"</pre>
```

The quant\_type column displays the type of data each row contains: astrometry (radec or seppa), or radial velocity (rv). For astrometry, quant1 column contains right ascension or separation, and the quant2 column contains declination or position angle. For rv data, quant1 contains radial velocity data in  $\rm km/s$ , while quant2 is filled with nan to preserve the data structure. The table contains each respective error column.

We can now initialize the Driver class. MCMC samplers take time to converge to absolute maxima in parameter space, and the more parameters we introduce, the longer we expect it to take.

# **Format Data**

In this example we use data from RadVel, so we want to change format to use it in orbitize.

```
[2]: # From P to sma:
    system_mass = 1 # [Msol]
    system_mass_err = 0.01 # [Msol]
    per_yr = per1 / 365.25
    pdf_msys = np.random.normal(system_mass, system_mass_err, size=len_pdf)
    sma_prior = (per_yr^{**}2 * pdf_msys) ** (1 / 3)
    # ecc from RadVel parametrization
    ecc_prior = sesinw1**2 + secosw1**2
    # little omega, i.e. argument of the periastron, from RadVel parametrization
    w_prior = (
        np.arctan2(sesinw1, secosw1) + np.pi
    ) # +pi bc of RadVel vs Orbitize convention
    w_prior[w_prior >= np.pi] = w_prior[w_prior >= np.pi] - 2 * np.pi
    def Tc_to_Tp(tc, per, ecc, omega): # stolen from radvel
        f = np.pi / 2 - omega
        ee = 2 * np.arctan(np.tan(f / 2) * np.sqrt((1 - ecc) / (1 + ecc)))
        tp = tc - per / (2 * np.pi) * (ee - ecc * np.sin(ee))
        return tp
    # tau, i.e. fraction of elapsed time of node passage, from RadVel parametrization
    tp1 = Tc_to_Tp(tc1, per1, ecc_prior, w_prior)
    tau_prior = basis.tp_to_tau(tp1, 55000, per1)
    # m1 prior
    K_0 = 28.4329
    m1sini_prior = (
        k1
         / K_0
        * np.sqrt(1.0 - ecc_prior**2.0)
        * pdf_msys ** (2.0 / 3.0)
        * per_yr ** (1 / 3.0)
    ) * 1e-3
    sini_prior = priors.SinPrior()
    i_prior_samples = sini_prior.draw_samples(len_pdf)
    m1_prior = m1sini_prior / np.sin(i_prior_samples)
```

We now have the orbital parameters that are accessible with RV: SMA, eccentricity, argument of periastron, and tau. Plus, given that we obtained Msini from RV, we draw random values for the inclination to get correlated values for m1 and inc. We thus end in this case with a correlated 6-parameter set.

# **Initialize Priors**

We initilize the System object to initialize the priors

```
[3]: # Initialize System object which stores data & sets priors
    data_table = read_input.read_file("{}/test_val.csv".format(DATADIR)) # read data
    num_secondary_bodies = 1
    system_orbitize = system.System(
        num_secondary_bodies,
        data_table,
        system_mass,
        1e1,
        mass_err=system_mass_err,
        plx_err=0.1,
        tau_ref_epoch=55000,
        fit_secondary_mass=True,
    )
    param_idx = system_orbitize.param_idx
    print(param_idx)
    {'sma1': 0, 'ecc1': 1, 'inc1': 2, 'aop1': 3, 'pan1': 4, 'tau1': 5, 'plx': 6, 'gamma_defrv
     →': 7, 'sigma_defrv': 8, 'm1': 9, 'm0': 10}
```

Let's initilize the KDE prior object with the default bandwidth.

```
[4]: from scipy.stats import gaussian_kde
    # The values go into a matrix
    total_params = 6
    values = np.empty((total_params, len_pdf))
    values[0, :] = sma_prior
    values[1, :] = ecc_prior
    values[2, :] = i_prior_samples
    values[3, :] = w_prior
    values[4, :] = tau_prior
    values[5, :] = m1_prior
    kde = gaussian_kde(
        values, bw_method=None
       # None indicates that the KDE bandwidth is set to default
    kde_prior_obj = priors.KDEPrior(
        kde, total_params
    ) # ,bounds=bounds_priors,log_scale_arr=[False,False,False,False,False,False])
    system_orbitize.sys_priors[
        param_idx["sma1"]
    ] = kde_prior_obj  # priors.GaussianPrior(np.mean(sma_prior), np.std(sma_prior))#priors.
     →KDEPrior(gaussian_kde(values[0,:], bw_method=None),1)#kde_prior_obj#
    system_orbitize.sys_priors[
        param_idx["ecc1"]
    ] = kde_prior_obj # priors.GaussianPrior(np.mean(ecc_prior), np.std(ecc_prior))#kde_
                                                                                (continues on next page)
```

(continues on next page)

```
→prior_obj#priors.GaussianPrior(np.mean(pdf_fromRadVel['e1']), np.std(pdf_fromRadVel['e1']))#priors.KDEPrior(gaussian_kde(values[1,:], bw_method=None),1)#kde_prior_obj
system_orbitize.sys_priors[param_idx["inc1"]] = kde_prior_obj
system_orbitize.sys_priors[
    param_idx["aop1"]
] = kde_prior_obj # priors.GaussianPrior(np.mean(w_prior), 0.1)#kde_prior_obj#kde_prior_
→ obj
system_orbitize.sys_priors[
    param_idx["tau1"]
] = kde_prior_obj # priors.GaussianPrior(np.mean(tau_prior), 0.01)#np.std(tau_prior))
    →#kde_prior_obj#kde_prior_obj#priors.KDEPrior(gaussian_kde(values[4,:], bw_method=None),
    →1)#kde_prior_obj
system_orbitize.sys_priors[-2] = kde_prior_obj
```

We can plot the KDE fit against the actual distribution to see if the selected bandwidth is adecuate for the data.

```
[5]: fig, axs = plt.subplots(2, 3, figsize=(11, 6))
     # sma
    axs[0, 0].hist(
        kde.resample(len_pdf)[0, :],
        density=True,
         color="b".
        label="Posterior Draw",
         stacked=True,
    )
    axs[0, 0].hist(
        sma_prior,
         100,
        density=True,
        color="r",
        histtype="step",
        label="Prior Draw",
        stacked=True,
    ) # color='r')#
    axs[0, 0].set_xlabel("SMA [AU]")
    axs[0, 0].set_yticklabels([])
    # ecc
    axs[0, 1].hist(
        kde.resample(len_pdf)[1, :],
         100.
        density=True,
        color="b",
        label="Posterior Draw",
         stacked=True,
    axs[0, 1].hist(
        ecc_prior,
         100,
        density=True,
        color="r",
        histtype="step",
```

```
label="Prior Draw",
    stacked=True,
) # color='r')#
axs[0, 1].set_xlabel("eccentricity")
axs[0, 1].set_yticklabels([])
# mass
masskde_arr = kde.resample(len_pdf)[-1, :]
masskde_arr = masskde_arr[masskde_arr < 0.003]</pre>
m1_prior_draw = m1_prior[m1_prior < 0.003]</pre>
axs[1, 0].hist(
    masskde_arr, 100, density=True, color="b", label="Posterior Draw", stacked=True
axs[1, 0].hist(
    (m1_prior_draw),
    100,
    density=True,
    color="r",
    histtype="step",
    label="Prior Draw",
    stacked=True,
) # color='r')#
axs[1, 0].set_xlabel("$Mass_b$ [$M_{Jup}$]")
axs[1, 0].set_yticklabels([])
# inc
axs[1, 1].hist(
    np.rad2deg(kde.resample(len_pdf)[2, :]),
    100.
    density=True,
    color="b",
    label="Posterior Draw",
    stacked=True,
axs[1, 1].hist(
    np.rad2deg(i_prior_samples),
    100,
    density=True,
    color="r",
    histtype="step",
    label="Prior Draw",
    stacked=True,
) # color='r')#
axs[1, 1].set_xlabel("inclination [deg]")
axs[1, 1].set_yticklabels([])
axs[1, 2].hist(
    np.rad2deg(kde.resample(len_pdf)[3, :]),
    100,
    density=True,
    color="b",
    label="Posterior Draw",
    stacked=True,
)
```

```
axs[1, 2].hist(
         np.rad2deg(w_prior),
         100,
         density=True,
         color="r",
         histtype="step",
         label="Prior Draw",
         stacked=True,
     ) # color='r')#
     axs[1, 2].set_xlabel("Arg. of periastron [deg]")
     axs[1, 2].set_yticklabels([])
     axs[0, 1].legend(bbox_to_anchor=(1.25, 1), loc="upper left", ncol=1)
     axs[0, 2].axis("off")
[5]: (0.0, 1.0, 0.0, 1.0)
                                                                                    Posterior Draw
                                                                                    Prior Draw
        3.70
                3.75
                       3.80
                                                     0.1
                                                            0.2
                 SMA (AU)
                                                    eccentricity
           0.000
                  0.001
                         0.002
                                0.003
                                                       100
                                                                  200
                                                                             -200
                                                                                   -100
                                                                                                 100
                                                                                                       200
                                             0
                                                            150
                                                                                           0
                                                  inclination [deg]
                                                                                 Arg. of periastron [deg]
                Mass_b [M_{Jup}]
```

As seen from the plot produced above, m1 is not well fit by the KDE; the bandwidth selected (we selected the default) is too broad and the lower bound of the mass is not well reproduced.

#### Select the KDE bandwidth

A temptation to fix the problem presented above, the oversmoothing of the data, would be to pick a very narrow bandwith, one that reproduces perfectly the data. However, the data from our posterior chains is finite, and contains throughout the distribution peaks and valleys that would introduce artifacts in the KDE fit contaminating the prior probabilities.

```
[6]: numtry_prior = 20
    sma_arr = np.mean(sma_prior) * np.linspace(0.98, 1.02, numtry_prior)
     # Initialize KDE with default bandwidth
    kde1 = gaussian_kde(
        values, bw_method=None
    ) # None indicates that the KDE bandwidth is set to default
     # Initialize KDE with narrow bandwidth
    bw2 = 0.15
    kde2 = gaussian_kde(values, bw_method=bw2)
    lnprior_arr1 = np.zeros((numtry_prior))
    lnprior_arr2 = np.zeros((numtry_prior))
    for idx_prior, sma in enumerate(sma_arr):
        lnprior_arr1[idx_prior] = kde1.logpdf(
             Ε
                 np.mean(ecc_prior),
                 np.pi / 2,
                 np.mean(w_prior),
                 np.mean(tau_prior),
                 np.mean(m1_prior),
             ]
        )
        lnprior_arr2[idx_prior] = kde2.logpdf(
                 sma.
                 np.mean(ecc_prior),
                 np.pi / 2,
                 np.mean(w_prior),
                 np.mean(tau_prior),
                 np.mean(m1_prior),
             ]
        )
    plt.figure(100)
    plt.plot(sma_arr, lnprior_arr1, label="KDE BW = Default")
    plt.plot(sma_arr, lnprior_arr2, label="KDE BW = {} (narrow)".format(bw2))
    plt.xlabel("SMA [AU]")
    plt.ylabel("log-prior probability")
    plt.legend(loc="upper right") # , fontsize='x-large')
     /var/folders/y8/lw5f1dcj04g4txq4y2znyc200000gn/T/ipykernel_15758/100748084.py:15:
     →DeprecationWarning: Conversion of an array with ndim > 0 to a scalar is deprecated,
     →and will error in future. Ensure you extract a single element from your array before.
     →performing this operation. (Deprecated NumPy 1.25.)
      lnprior_arr1[idx_prior] = kde1.logpdf(
                                                                                 (continues on next page)
```

Chapter 2. User Guide:

```
/var/folders/y8/lw5f1dcj04g4txq4y2znyc200000gn/T/ipykernel_15758/100748084.py:25:_

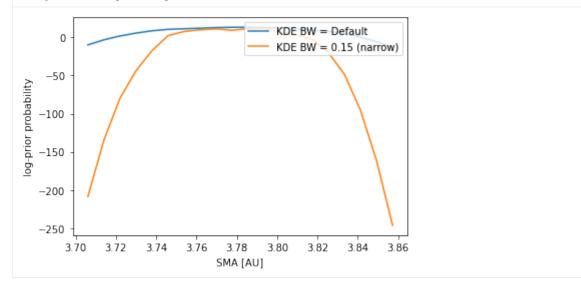
→DeprecationWarning: Conversion of an array with ndim > 0 to a scalar is deprecated,

→ and will error in future. Ensure you extract a single element from your array before

→ performing this operation. (Deprecated NumPy 1.25.)

lnprior_arr2[idx_prior] = kde2.logpdf(
```

[6]: <matplotlib.legend.Legend at 0x7f9fe2a17d30>



The plot above ilustrates that we cannot go arbitrarily narrow for the KDE bandwidth.

A way of choosing the KDE bandwidth is: 1. Pick an acceptable change in the median and 68th interval limits of the KDE fit w.r.t the actual posterior distribution. This will set a maximum acceptable bandwidth. 2. Pick an acceptable variation of the log-prior probability when evaluating the priors for a SMA around the prior median SMA. This will set a minimum acceptable bandwidth.

For this we will loop over a set of bandwidths computing the median and 68th interval limits, and for each bandwidth we will compute the variation of log-prior with a set of SMAs around the prior median SMA: we will fit a Gaussian and the standard deviation of the residuals to the fit will be our cost function.

```
def gaussian(x, amp, cen, wid, bias):
    return amp * np.exp(-((x - cen) ** 2) / wid) + bias

numtry_bw = 10
    kde_bw_arr = np.linspace(0.05, 0.1, numtry_bw)
    diff_mean_arr = np.zeros((numtry_bw))
    diff_p_arr = np.zeros((numtry_bw))
    diff_m_arr = np.zeros((numtry_bw))
    res_fit_prior2gauss = np.zeros((numtry_bw))

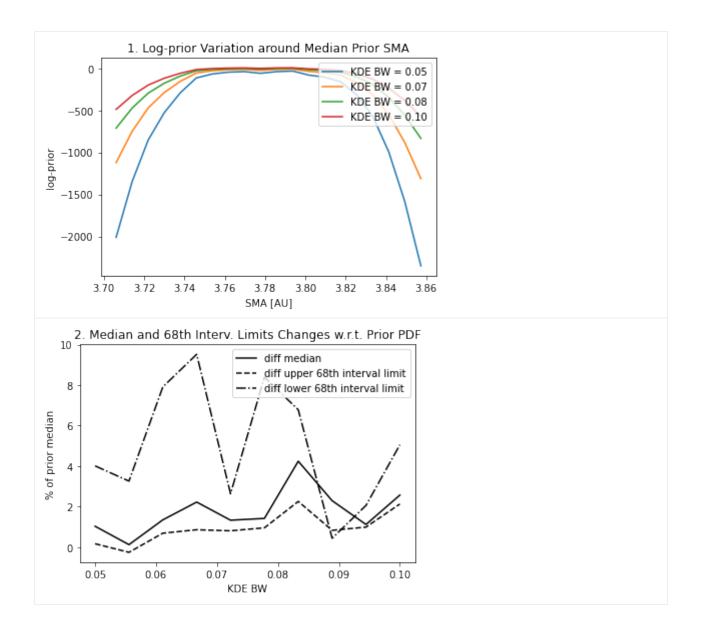
numtry_prior = 20
    sma_arr = np.mean(sma_prior) * np.linspace(0.98, 1.02, numtry_prior)

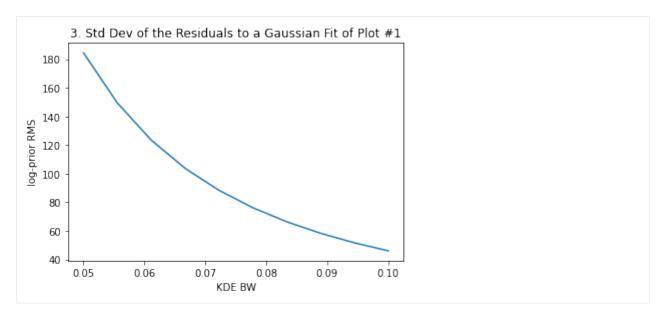
for idx_bw, kde_bw in enumerate(kde_bw_arr):

    (continues on next page)
```

```
kde = gaussian_kde(values, bw_method=kde_bw)
# Check pdf
lnprior_arr = np.zeros((numtry_prior))
for idx_prior, sma in enumerate(sma_arr):
    lnprior_arr[idx_prior] = kde.logpdf(
        Ε
            sma,
            np.mean(ecc_prior),
            np.pi / 2,
            np.mean(w_prior),
            np.mean(tau_prior),
            np.mean(m1_prior),
        ]
    )
# Quarentiles comparison
masskde_arr = kde.resample(len_pdf)[5, :]
masskde_arr = masskde_arr[masskde_arr < 0.004]</pre>
masskde_quantiles = np.quantile(
    masskde_arr * 1000, [(1 - 0.68), 0.5, 0.5 + (1 - 0.68)]
)
massprior_quantiles = np.quantile(
    m1_prior_draw * 1000, [(1 - 0.68), 0.5, 0.5 + (1 - 0.68)]
diff_mean_arr[idx_bw] = (
    masskde_quantiles[1] - massprior_quantiles[1]
) / massprior_quantiles[1]
diff_p_arr[idx_bw] = (
    (masskde_quantiles[1] - masskde_quantiles[0])
    - (massprior_quantiles[1] - massprior_quantiles[0])
) / massprior_quantiles[1]
diff_m_arr[idx_bw] = np.abs(
    (
        (masskde_quantiles[2] - masskde_quantiles[1])
        - (massprior_quantiles[2] - massprior_quantiles[1])
    / massprior_quantiles[1]
)
# fit to Gaussian
n = len(sma\_arr)
mean = np.mean(sma_prior)
sigma = 0.04 # note this correction
best_vals, covar = curve_fit(
    gaussian,
    sma_arr,
    lnprior_arr,
    ]=0q
        np.abs(np.max(lnprior_arr) - np.min(lnprior_arr)),
        np.mean(sma_prior),
        0.04,
```

```
np.mean(lnprior_arr) * 4,
            ],
        # [np.mean(lnprior_arr),mean,sigma])
        gauss_fit = gaussian(sma_arr, *best_vals)
        if idx_bw % 3 == 0:
            plt.figure(101)
            plt.plot(sma_arr, lnprior_arr, label="KDE BW = {:.2f}".format(kde_bw))
        res_fit_prior2gauss[idx_bw] = np.std(gauss_fit - lnprior_arr)
    plt.figure(101)
    plt.legend(loc="upper right")
    plt.xlabel("SMA [AU]")
    plt.ylabel("log-prior")
    plt.title("1. Log-prior Variation around Median Prior SMA")
    plt.figure(301)
    plt.plot(kde_bw_arr, diff_mean_arr * 100, color="k", label="diff median")
    plt.plot(
        kde_bw_arr,
        diff_p_arr * 100,
        color="k",
        linestyle="--",
        label="diff upper 68th interval limit",
    plt.plot(
        kde_bw_arr,
        diff_m_arr * 100,
        color="k",
        linestyle="-.",
        label="diff lower 68th interval limit",
    )
    plt.xlabel("KDE BW")
    plt.ylabel("% of prior median")
    plt.legend(loc="upper right") # , fontsize='x-large')
    plt.title("2. Median and 68th Interv. Limits Changes w.r.t. Prior PDF")
    plt.figure(302)
    plt.plot(kde_bw_arr, res_fit_prior2gauss, label="")
    plt.xlabel("KDE BW")
    plt.ylabel("log-prior RMS")
    plt.title("3. Std Dev of the Residuals to a Gaussian Fit of Plot #1")
    /var/folders/y8/lw5f1dcj04g4txq4y2znyc200000gn/T/ipykernel_15758/2442406081.py:24:
     →DeprecationWarning: Conversion of an array with ndim > 0 to a scalar is deprecated,
     →and will error in future. Ensure you extract a single element from your array before.
     →performing this operation. (Deprecated NumPy 1.25.)
      lnprior_arr[idx_prior] = kde.logpdf(
    /Users/bluez3303/miniconda3/envs/python3.10/lib/python3.10/site-packages/scipy/optimize/_
     →minpack_py.py:833: OptimizeWarning: Covariance of the parameters could not be estimated
      warnings.warn('Covariance of the parameters could not be estimated',
[7]: Text(0.5, 1.0, '3. Std Dev of the Residuals to a Gaussian Fit of Plot #1')
```





The above plots #2 and #3 give us the information to select the most adecuate KDE bandwidth. For instance, if we decide that a ~3-4% difference in the median and 68th interval limits is acceptable, that sets a maximum bandwidth of ~0.9

To make a choice for the log-prior probability variation in the SMA range we picked, we can see what variation does the log-likelihood present for the same SMA range. For the same SMA range, we compute the log-likelihood of our model and see the peak-to-valley to assess what variation we want to allow for a narrow bandwidth.

Once you've chosen the bandwidth that best fits your posteriors you can use them as priors for a MCMC fit. Link to MCMC tutotial.

# 2.2.12 Fitting in different orbital bases

In this tutorial, we show how one can perform orbit-fits in different coordinate bases amongst the ones supported by orbitize. Currently fitting in different bases is only supported in MCMC, so we will use MCMC to perform an orbit-fit in an orbital basis distinct from the default one. For a general introduction to MCMC, be sure to check out the MCMC Introduction tutorial first!

# The "standard" and "XYZ" bases

The default way to define an orbit in orbitize is through what we call the 'standard basis', which consists of eight parameters: semi-major axis (sma), eccentricity (ecc), inclination (inc), argument of periastron (aop), position angle of the nodes (pan), epoch of periastron expressed as a fraction of the period past a reference epoch (tau), parallax (plx) and total system mass (mtot). Each orbital element has an associated default prior; to see how to explore and modify these priors check out the Modifying priors tutorial.

An alternative way to define an orbit is through its position and velocity components in XYZ space for a given epoch; we will call this the 'XYZ basis'. The orbit is thus defined with the array  $(x, y, z, \dot{x}, \dot{y}, \dot{z}, \operatorname{plx}, \operatorname{mtot})$ , with position coordinates measured in AU and velocity components in km s<sup>-1</sup>. In this basis, the sky-plane coordinates (x, y) are the separations of the planet relative to the primary, with the positive x and y directions coinciding with the positive RA and Dec directions, respectively. The z direction is the line-of-sight coordinate, such that movement in the positive z direction causes a redshift. The default priors are uniform all uniform.

# Setting up Sampler in the XYZ basis

The easiest way to run an orbit-fit in an alternative orbital basis in orbitize is through the orbitize.driver.Driver interface. The process is exactly like initializing a regular Driver object, but setting the fitting\_basis keyword to 'XYZ':

```
[1]: import numpy as np
    import orbitize
    from orbitize import driver
    import multiprocessing as mp
    filename = "{}xyz_test_data.csv".format(orbitize.DATADIR) # a file with input in radec_
     ⇒since rn it only works for that
    # system parameters
    num_secondary_bodies = 1
    system_mass = 1.75 \# [Msol]
    plx = 51.44 \# [mas]
    mass\_err = 0.05 \# [Msol]
    plx_err = 0.12 # [mas]
    # MCMC parameters
    num\_temps = 5
    num_walkers = 20
    num_threads = mp.cpu_count() # or a different number if you prefer
    my_driver = driver.Driver(
         filename, 'MCMC', num_secondary_bodies, system_mass, plx, mass_err=mass_err, plx_
     →err=plx_err,
        mcmc_kwargs={'num_temps': num_temps, 'num_walkers': num_walkers, 'num_threads': num_

→threads
},
         system_kwargs={'fitting_basis': 'XYZ'}
    )
    s = my_driver.sampler
    Converting ra/dec data points in data_table to sep/pa. Original data are stored in input_
     →table.
```

# (Properly) initializing walkers in the XYZ basis

In the standard basis at this point we would be ready to use the s.run\_sampler method to start the sampling, but with the XYZ basis we have to make sure that all our walkers are initialized in a valid region of parameter space. This is because randomly generated values of  $(x, y, z, \dot{x}, \dot{y}, \dot{z})$  can result in unbound, invalid orbits with, for example, negative eccentricities (which is not cool). This can be easily corrected with the s.validate\_xyz\_positions method:

(continues on next page)

```
result = super().__array_ufunc__(function, method, *arrays, **kwargs)
    After this is done, the sampler can be run and the results saved normally:
[3]: total_orbits = 600 # number of steps x number of walkers (at lowest temperature)
    burn_steps = 10 # steps to burn in per walker
    thin = 2 # only save every 2nd step
    s.run_sampler(total_orbits, burn_steps=burn_steps, thin=thin)
    s.results.save_results('my_posterior.hdf5')
    /data/user/sblunt/miniconda3/envs/python3.7/lib/python3.7/site-packages/astropy/table/
     →column.py:1020: RuntimeWarning: invalid value encountered in greater
      result = getattr(super(), op)(other)
    /home/sblunt/Projects/orbitize/orbitize/kepler.py:112: RuntimeWarning: invalid value
     →encountered in sqrt
      tanom = 2.*np.arctan(np.sqrt((1.0 + ecc)/(1.0 - ecc))*np.tan(0.5*eanom))
    Starting Burn in
    /data/user/sblunt/miniconda3/envs/python3.7/lib/python3.7/site-packages/astropy/table/
     →column.py:1020: RuntimeWarning: invalid value encountered in greater
      result = getattr(super(), op)(other)
    /home/sblunt/Projects/orbitize/orbitize/kepler.py:112: RuntimeWarning: invalid value
     →encountered in sqrt
      tanom = 2.*np.arctan(np.sqrt((1.0 + ecc)/(1.0 - ecc))*np.tan(0.5*eanom))
    /data/user/sblunt/miniconda3/envs/python3.7/lib/python3.7/site-packages/astropy/table/
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      result = getattr(super(), op)(other)
    /home/sblunt/Projects/orbitize/orbitize/kepler.py:112: RuntimeWarning: invalid value_
     →encountered in sqrt
      tanom = 2.*np.arctan(np.sqrt((1.0 + ecc)/(1.0 - ecc))*np.tan(0.5*eanom))
    /data/user/sblunt/miniconda3/envs/python3.7/lib/python3.7/site-packages/astropy/table/

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     →column.py:1020: RuntimeWarning: invalid value encountered in greater
      result = getattr(super(), op)(other)
    /home/sblunt/Projects/orbitize/orbitize/kepler.py:112: RuntimeWarning: invalid value_
     ⊶encountered in sqrt
```

/data/user/sblunt/miniconda3/envs/python3.7/lib/python3.7/site-packages/astropy/units/

→quantity.py:481: RuntimeWarning: invalid value encountered in sqrt

eanom = np.arccos(cos\_eanom)

```
tanom = 2.*np.arctan(np.sqrt((1.0 + ecc)/(1.0 - ecc))*np.tan(0.5*eanom))
/data/user/sblunt/miniconda3/envs/python3.7/lib/python3.7/site-packages/astropy/table/
→column.py:1020: RuntimeWarning: invalid value encountered in greater
  result = getattr(super(), op)(other)
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→encountered in sqrt
  tanom = 2.*np.arctan(np.sqrt((1.0 + ecc)/(1.0 - ecc))*np.tan(0.5*eanom))
10/10 steps of burn-in complete
Burn in complete. Sampling posterior now.
/data/user/sblunt/miniconda3/envs/python3.7/lib/python3.7/site-packages/astropy/table/
→column.py:1020: RuntimeWarning: invalid value encountered in greater
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  tanom = 2.*np.arctan(np.sqrt((1.0 + ecc)/(1.0 - ecc))*np.tan(0.5*eanom))
```

```
/data/user/sblunt/miniconda3/envs/python3.7/lib/python3.7/site-packages/astropy/table/

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/data/user/sblunt/miniconda3/envs/python3.7/lib/python3.7/site-packages/astropy/table/
                                                                          (continues on next page)
```

```
→column.py:1020: RuntimeWarning: invalid value encountered in greater
  result = getattr(super(), op)(other)
/home/sblunt/Projects/orbitize/orbitize/kepler.py:112: RuntimeWarning: invalid value_
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  result = getattr(super(), op)(other)
/home/sblunt/Projects/orbitize/orbitize/kepler.py:112: RuntimeWarning: invalid value_
→encountered in sqrt
  tanom = 2.*np.arctan(np.sqrt((1.0 + ecc)/(1.0 - ecc))*np.tan(0.5*eanom))
30/30 steps completed
Run complete
```

# Loading and converting results

You can load the results as you normally would. The orbit posteriors are saved in the results.post attribute, and the basis you used for the fit in the results.fitting\_basis attribute:

```
[4]: myResults = orbitize.results.Results() # create empty Results object
                myResults.load_results('my_posterior.hdf5')
                print('The used basis for the fit was ', myResults.fitting_basis)
                print('The posteriors are ', myResults.post)
                Converting ra/dec data points in data_table to sep/pa. Original data are stored in input_
                  →table.
                The used basis for the fit was XYZ
                The posteriors are [[-1.55895340e+01 -3.20352269e+01 9.38119986e+00 ... -7.46195838e-03
                           5.15299823e+01 1.72400897e+00]
                    [-1.56081092e+01 -3.20562773e+01  1.49810951e+00  ...  8.53387015e-02
                           5.13883909e+01 1.73579787e+00]
                     \begin{bmatrix} -1.55612462e + 01 & -3.20801251e + 01 & -2.96308303e - 01 & \dots & 7.18261775e - 01 \end{bmatrix} 
                           5.14206555e+01 1.76608956e+00]
                    [-1.55671475e+01 -3.20089823e+01  3.62094122e+01  ...  3.72598271e-01
                           5.15487665e+01 1.74432532e+00]
                     \begin{bmatrix} -1.55794884e + 01 & -3.20303979e + 01 & 2.75912018e + 01 & \dots & -2.40407388e - 01 \end{bmatrix} 
                           5.14895599e+01 1.78876637e+00]
                     \begin{bmatrix} -1.55837449 \\ e+01 & -3.20532712 \\ e+01 & 1.93382300 \\ e+01 & \dots & -1.24185021 \\ e-01 & \dots & 
                           5.14598433e+01 1.80483891e+00]]
```

Let's convert back to the good old standard basis:

```
[5]: xyz_posterior = myResults.post
    standard_posterior = myResults.system.basis.to_standard_basis(xyz_posterior)
    print('My posterior in standard basis is ', standard_posterior)
    My posterior in standard basis is [[ 7.89543529 6.92670011
                                                                                         0.
                                                                        4.84646336 ...
     \rightarrow 36153132 -6.60018059
         1.52254522]
      [ 1.00000429
                      1.00000659
                                    0.96238341 ... 1.00053929
                                                                   1.00014326
        0.9999977 ]
                                    0.79516331 ... 1.46937244 2.46248534
      Γ 1.43499876
                     1.22274162
         1.23409917]
      [-15.56714749 - 32.00898233 \ 36.20941224 \dots \ 0.37259827 \ 51.54876654
         1.74432532]
      [-15.57948843 - 32.03039793 \ 27.59120179 \dots -0.24040739 \ 51.48955988
         1.788766371
      [-15.58374489 - 32.0532712 \quad 19.33822997 \dots -0.12418502 \quad 51.45984325
         1.80483891]]
```

And we're done! Enjoy the XYZ basis.

# 2.2.13 Working with the Hipparcos Intermediate Astrometric Data (IAD)

Sarah Blunt (2021)

The Hipparcos IAD are notoriously annoying to work with, and several methods have been developed for incorporating them into orbit-fits. In this tutorial, we'll take you through the method outlined in Nielsen et al. (2020). In the near future, we'll add in other algorithms (in particular, that of Brandt et al.).

The purpose of this tutorial is not to convince you that one method is better than the other, but rather to teach you to use the method of Nielsen et al. I recommend reading through Section 3.1 of Nielsen et al. (2020) before diving into this tutorial.

If you want to skip right to "what's the syntax for doing an orbit-fit with the IAD," I suggest checking out the beta Pictoris end-to-end test I coded up here.

This tutorial will take you through: - Obtaining the IAD for your object(s). - Refitting the IAD (i.e. re-doing the fit the Hipparcos team did), which allows you to evaluate whether they are suitable for orbit-fitting. - Incorporating the IAD into your orbit-fit.

# Part 1: Obtaining the IAD

orbitize! assumes the IAD are an updated version of the van Leeuwen (2007) re-reduction. You can learn about the data here, and download them here. Note that this download will take ~350 MB of space; if you prefer to to keep the data necessary for a few fits, you can download all the data and remove unnecessary files (or just send me an email). We've provided the IAD files for a few representative systems in this repository.

NOTE: if you prefer to use the DVD files, orbitize! can handle those as well. However, be warned that these files do not mark rejected scans.

The good news is that the hard part is actually getting the data. Once you have the file, orbitize! can read it in its raw form.

# Part 2: Refitting the IAD

Once you have the IAD, the next step is to convince yourself that they're suitable for orbit-fitting (i.e. free of transcription errors, etc). Here's a handy function to do this (this should take a few mins to run, but if it's taking much longer, you can decrease the number of steps). This code reproduces the test at the end of Section 3.1 of Nielsen et al. (2020), so check that out for more information.

```
import orbitize
import os
from datetime import datetime

from orbitize.hipparcos import nielsen_iad_refitting_test

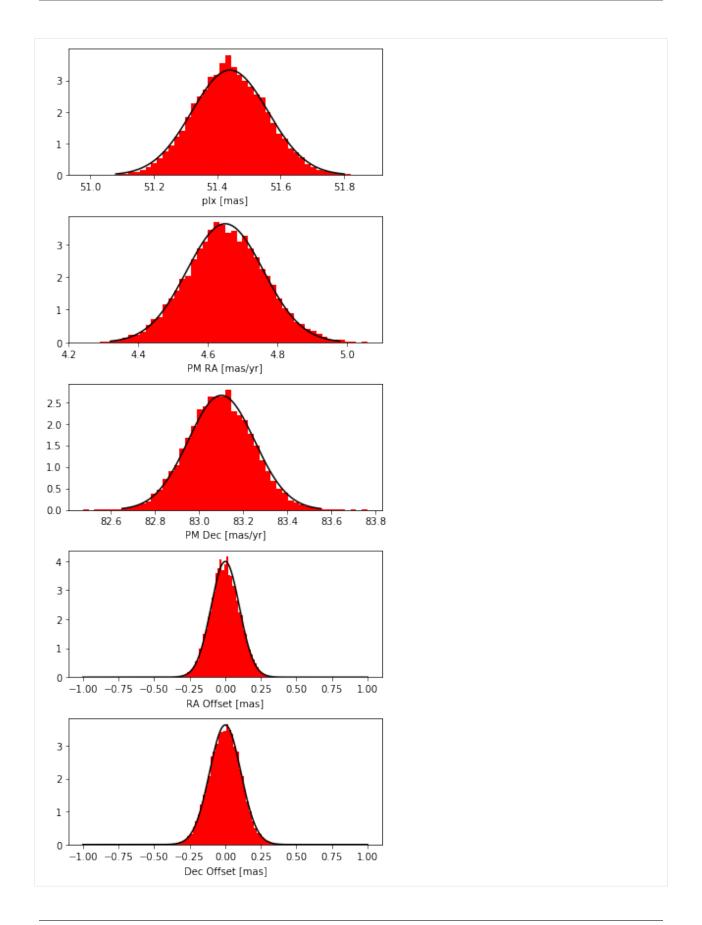
# The Hipparcos ID of your target. Available on Simbad.
hip_num = "027321"

# Name/path for the plot this function will make
saveplot = "bPic_IADrefit.png"

# Location of the Hipparcos IAD file.
IAD_file = "{}H{}.d".format(orbitize.DATADIR, hip_num)

# These `emcee` settings are sufficient for the 5-parameter fits we're about to run,
```

```
although I'd probably run it for 5,000-10,000 steps if I wanted to publish it.
burn_steps = 100
mcmc\_steps = 100
start = datetime.now()
# run the fit
nielsen_iad_refitting_test(
    IAD_file,
    hip_num=hip_num,
    saveplot=saveplot,
    burn_steps=burn_steps,
    mcmc_steps=mcmc_steps,
end = datetime.now()
duration_mins = (end - start).total_seconds() / 60
print("Done! This fit took {:.1f} mins on my machine.".format(duration_mins))
# If you don't want to save the plot, you can run this line to remove it
_ = os.system("rm {}".format(saveplot))
Go get a coffee. This will take a few mins! :)
Starting burn-in!
Starting production chain!
Done! This fit took 3.1 mins on my machine.
```



# Part 3: Using the IAD in your Orbit-fit

Congrats, you've now reproduced a Hipparcos fit! The last thing to do is actually run your orbit-fit. Here's an example, also using the Gaia astrometric point from eDR3. This code snippet essentially repeats the beta Pictoris end-to-end test I coded up here.

```
[2]: import os.path
    import orbitize
    from orbitize import read_input, hipparcos, gaia, system, priors, sampler
    As with most `orbitize!` fits, we'll start by reading in our data file. The
    Hipparcos data file is kept separate; your main data file only needs to contain
    the relative astrometry and RVs you're using in your fit.
    input_file = os.path.join(orbitize.DATADIR, "betaPic.csv")
    data_table = read_input.read_file(input_file)
    Next, we'll instantiate a `HipparcosLogProb` object.
    num_secondary_bodies = 1 # number of planets/companions orbiting your primary
    hipparcos_number = "027321" # (can look up your object's Hipparcos ID on Simbad)
    hipparcos_filename = os.path.join(
        orbitize.DATADIR, "H027321.d"
    ) # location of your IAD data file
    betaPic_Hip = hipparcos.HipparcosLogProb(
        hipparcos_filename, hipparcos_number, num_secondary_bodies
    )
    Next, instantiate a `GaiaLogProb` object.
    betapic_edr3_number = 4792774797545800832
    betaPic_Gaia = gaia.GaiaLogProb(betapic_edr3_number, betaPic_Hip, dr="edr3")
    Next, we'll instantiate a `System` object, a container for all the information
    relevant to the system you're fitting.
    m0 = 1.75 # median mass of primary [M_sol]
    plx = 51.5 \# [mas]
    fit_secondary_mass = True # Tell orbitize! we want to get dynamical masses
    # (not possible with only relative astrometry).
    mass_err = 0.01 # we'll overwrite these in a sec
    plx_err = 0.01
    betaPic_system = system.System(
        num_secondary_bodies,
        data_table,
```

(continues on next page)

```
m0,
   plx,
   hipparcos_IAD=betaPic_Hip,
   gaia=betaPic_Gaia,
   fit_secondary_mass=fit_secondary_mass,
   mass_err=mass_err,
   plx_err=plx_err,
)
If you'd like to change any priors from the defaults (given in Blunt et al. 2020),
do it like this:
# set uniform parallax prior
plx_index = betaPic_system.param_idx["plx"]
betaPic_system.sys_priors[plx_index] = priors.UniformPrior(plx - 1.0, plx + 1.0)
# set uniform primary mass prior
m0_index = betaPic_system.param_idx["m0"]
betaPic_system.sys_priors[m0_index] = priors.UniformPrior(1.5, 2.0)
INFO: Query finished. [astroquery.utils.tap.core]
```

Finally, set up and run your MCMC!

```
[3]: """
    These are the MCMC parameters I'd use if I were publishing this fit.
     This would take a while to run (takes about a day on my machine).
    # num_threads = 50
    \# num_temps = 20
     # num_walkers = 1000
    # num_steps = 10000000 # n_walkers x n_steps_per_walker
     # burn_steps = 10000
    # thin = 100
    Here are some parameters you can use for the tutorial. These chains will not
    be converged.
    num_threads = 1
    num\_temps = 1
    num_walkers = 100
    num_steps = 10 # n_walkers x n_steps_per_walker
    burn_steps = 10
    thin = 1
    betaPic_sampler = sampler.MCMC(
        betaPic_system,
        num_threads=num_threads,
        num_temps=num_temps,
```

# [4]: # make corner plot

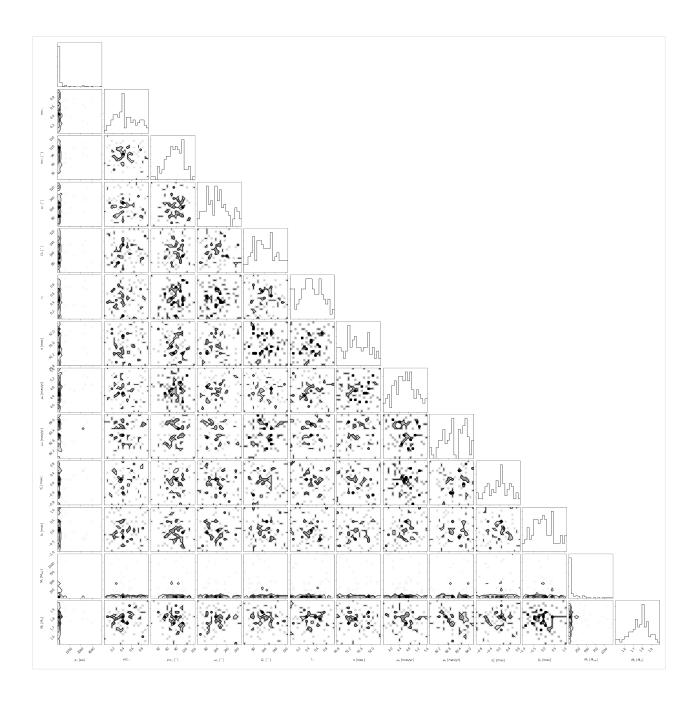
```
fig = betaPic_sampler.results.plot_corner()
WARNING:root:Too few points to create valid contours
```

(continues on next page)

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WARNING:root:Too few points to create valid contours WARNING:root:Too few points to create valid contours

```
WARNING:root:Too few points to create valid contours
```



# 2.3 Frequently Asked Questions

Here are some questions we get often. Please suggest more by raising an issue in the Github Issue Tracker.

# What does this orbital parameter mean?

We think the best way to understand the orbital parameters is to see how they affect the orbit visually. Play around with this interactive orbital elements notebook (you'll need to run on your machine).

# What is and how is it related to epoch of periastron?

We use to define the epoch of periastron as we do not know when periastron will be for many of our directly imaged planets. A detailed description of how is related to other quantities such as  $t_p$  is available:

# 2.3.1 $\tau$ and Time of Periastron

Here, we will discuss what exactly is  $\tau$ , the parameter orbitize! uses to parametrize the epoch of periastron, and how it is related to other quantities of the epoch of periastron in the literature.

#### Time of Periastron and Motivation for au

The time (or epoch) of periastron is an important quantity for describing an orbit. It defines when the two orbiting bodies are closest to one another (i.e., when a planet is closest to its star). In many papers in the literature, the epoch of periastron is described by  $t_p$ , which is literally a date at which periastron occurs. This is a very important date because we use this date to anchor our orbit in time.

The value of  $t_p$  is well constrained when we know we observed periastron, which is often the case for radial velocity or transiting exoplanets when the orbital periods are short and our data covers a full orbital period. In those cases, we know approximately when  $t_p$  should be in time, so it is easy to define prior bounds for it. However, in the case of direct imaging, many of our companions have orbital periods that are orders of magnitude larger than the current orbital coverage of the data where we do not really know if the next periastron is in years, decades, centuries, or even millennia. This is the motivation for  $\tau$ .

#### **Definition of** $\tau$

 $\tau$  is a dimentionless quantity between 0 and 1 defined with respect to a reference epoch  $t_{ref}$ . For a planet that has a  $t_p$  and an orbital period (P), then we define  $\tau$  as:

$$\tau = \frac{t_p - t_{ref}}{P}.$$

Because  $\tau$  is always between 0 and 1, it is easy to figure out the bounds of  $\tau$  whereas if the orbital period is highly uncertain, it may be difficult to put bounds on  $t_p$  that would encompass all allowable bound orbits.

# Relation to $t_p$

As seen in the above equation, it is relatively straightforward to covert between orbital parameter sets that use  $\tau$  and  $t_p$ . You just need to know the orbital period and reference epoch. In orbitize!, both the System class and the Results class has the attribute tau\_ref\_epoch which stores  $t_{ref}$ , so there should always be a convenient way to grab this number. By default, we use  $t_{ref} = 58849$  MJD.

One thing to note that is a given orbit has only a single valid  $\tau$ , but that an orbit can be defined by many  $t_p$ , since the orbit is periodic. Thus,  $t_p + P$  is another valid time of periastron.

We also provide some helper functions to covert between  $t_p$  and  $\tau$ 

```
[1]: import numpy as np
  import orbitize.basis

# How to get orbital period in the orbitize! standard basis
  sma = 9 # au, semi-major axis
  mtot = 1.2 # Solar masses, total mass
  period = np.sqrt(sma**3/mtot) # years, period

tau = 0.2
  tau_ref_epoch = 58849

# convert tau to tp
```

```
tp = orbitize.basis.tau_to_tp(tau, tau_ref_epoch, period)
print(tp)

# convert tp back to tau
tau2 = orbitize.basis.tp_to_tau(tp, tau_ref_epoch, period)
print(tau2)

# convert tau to tp, but pick the first tp after MJD = 0
tp_new = orbitize.basis.tau_to_tp(tau, tau_ref_epoch, period, after_date=0)
print(tp_new)

60649.50097715886
0.200000000000000002
6634.471662393138
```

# **Relation to Mean Anomaly**

The mean anomaly (M) of an orbit describes the current orbital phase of a planet. M goes from 0 to  $2\pi$ , and M = 0 means the planet is at periastron. Unlike  $t_p$  and  $\tau$  which describe the epoch of periastron, M describes the current position of the planet in its orbit.

To compute M of a planet at some time t, we have provided the following helper function:

```
[2]: # Use the orbit defined in the previous example

t = 60000 # time in MJD when we want to know the M of the particle

M = orbitize.basis.tau_to_manom(t, sma, mtot, tau, tau_ref_epoch)

print(M)

# now compute M for periastron
M_peri = orbitize.basis.tau_to_manom(tp, sma, mtot, tau, tau_ref_epoch)

print(M_peri)

5.829874251150844
1.3951473992034527e-15
```

#### Why is the default prior on inclination a sine prior?

Our goal with the default prior is to have an isotropic distribution of the orbital plane. To obtain this, we use inclination and position angle of the ascending node (PAN) to define the orbital plane. They actually coorespond to the two angles in a spherical coordinate system: inclinaion is the polar angle and PAN is the azimuthal angle. Becuase of this choice of coordinates, there are less orbital configurations near the poles (when inclination is near 0 or 180), so we use a sine prior to downweigh those areas to give us an isotropic distribution. A more detailed discussion of this is here:

# **2.3.2** Defining the orbital plane with i and $\Omega$

```
(Jason Wang, 2021)
```

Here we discuss how the orbit plane orientation is defined. We also encourage you to play around with this interactive orbital elements notebook to get a feel for the orbital elements. Also note that we refer to the values of the angles in degrees when discussing them but all angles are in radians in orbitize!.

Inclination (i) and Position angle of the Ascending Node (PAN;  $\Omega$ ) define the plane of the orbit in the sky. Inclination describes the tilt of the orbital plane relative to the plane in the sky, and PAN describes the rotation of the line of nodes, which is the intersection between the orbital plane and the plane of the sky.

An intuitive way to think about it is in terms of spherical coordinates. i is equivalent to  $\theta$  (range from 0 to 180 deg), and  $\Omega$  is equivalent to  $\varphi$  (range from 0 to 360 deg) in common spherical notation. The sphere in this case is tilted, with one of the poles pointed towards us.

# Why do we use a sine prior on inclination?

You might also hear about using a uniform prior in  $\cos(i)$ , which is an equivalent statement. We use a sine prior on inclination because our ultimate goal is to have an isotropic prior on the orbital plane, and due to how we choose coordiantes, this becomes a sine prior on inclination. We can go through some math/visual explaination, but the easiest is perhaps seeing the distribution of orbital plane orientations on a sphere. We use the orbital plane normal (perpendicular vector from the orbit plane) to define the orientation of the plane in 3D space. The normal points to a single point on a unit sphere. Isotropic distributions will uniformly cover the unit sphere.

In the exercise below, if we randomly draw orbital plane orientations using either an uniform or sine prior for inclination, you'll see that the uniform prior causes more points to cluster near the poles, whereas the sine prior is more uniform. Note that generally the edges of the sphere look darker merely due to a viewing angle effect.

```
[4]: import numpy as np
    import matplotlib.pylab as plt
    from orbitize.priors import UniformPrior, SinPrior
    %matplotlib inline
    def spherical_to_xyz(theta, phi, rho=1):
         Transformation with theta and phi in radians
        z = rho * np.cos(theta)
        x = rho * np.sin(theta) * np.cos(phi)
        y = rho * np.sin(theta) * np.sin(phi)
        return x,y,z
    fig = plt.figure()
    # try a uniform distribution in both
    ax1 = fig.add_subplot(121, projection='3d')
    inc_uni_prior = UniformPrior(0, np.pi)
    pan_uni_prior = UniformPrior(0, 2*np.pi)
    incs_uni = inc_uni_prior.draw_samples(2000)
    pan_uni = pan_uni_prior.draw_samples(2000)
    x_uni, y_uni, z_uni = spherical_to_xyz(incs_uni, pan_uni)
```

```
ax1.plot(x_uni, y_uni, z_uni, 'b.', alpha=0.1)
ax1.set_title("Uniform in inc")

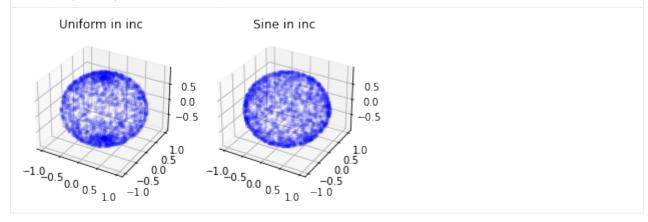
# try a sine distribution for inclination
ax2 = fig.add_subplot(122, projection='3d')

inc_sine_prior = SinPrior()

incs_sine = inc_sine_prior.draw_samples(2000)

x_uni, y_uni, z_uni = spherical_to_xyz(incs_sine, pan_uni)
ax2.plot(x_uni, y_uni, z_uni, 'b.', alpha=0.1)
ax2.set_title("Sine in inc")
```

# [4]: Text(0.5, 0.92, 'Sine in inc')



# What do the values of i mean?

Inclination in orbitize! is defined to go from 0 to 180 degrees. Some other places use -90 to 90 instead, but -90 to 0 is the same as 90 to 180. There are actual a couple of quick things to learn about the orbit from the value of inclination. Here is a summary:

- $i=0^\circ$ : orbit is face-on and body orbits counterclockwise in the sky (North-up, East-left)
- $0^{\circ} < i < 90^{\circ}$ : orbit is inclined and body orbits counterclockwise in the sky (North-up, East-left)
- $i=90^{\circ}$  : orbit is viewed edge-on
- $90^{\circ} < i < 180^{\circ}$ : orbit is inclined and body orbits clockwise in the sky (North-up, East-left)
- $i = 180^{\circ}$ : orbit is face-on and body orbits clockwise in the sky (North-up, East-left)

Here are some examples below:

```
[5]: from orbitize.kepler import calc_orbit

sma = 1
ecc, pan, aop, plx, mtot = 0, 0, 0, 1, 1
tau = 0.3

incs = np.radians([0, 45, 90, 135, 180])
```

```
all_{eps} = np.linspace(0, 365.25, 200)
arc_{eps} = np.linspace(0, 75, 75)
fig = plt.figure(figsize=(15,4))
for i, inc in enumerate(incs):
    ax = fig.add\_subplot(1, 5, i+1)
    all_ras, all_decs, _ = calc_orbit(all_eps, sma, ecc, inc, aop, pan, tau, plx, mtot,__
→tau_ref_epoch=0)
    ax.plot(all_ras, all_decs, 'k--', alpha=0.5)
    arc_ras, arc_decs, _ = calc_orbit(arc_eps, sma, ecc, inc, aop, pan, tau, plx, mtot,__
→tau_ref_epoch=0)
    ax.plot(arc_ras, arc_decs, 'b-', linewidth=2)
    ax.plot(arc_ras[-1], arc_decs[-1], 'bo', markersize=7)
    ax.set_title("i = {0:d} deg".format(int(np.degrees(inc))))
    ax.set_aspect("equal")
    ax.set_xlim([1.1, -1.1])
    ax.set_ylim([-1.1, 1.1])
    ax.set_xlabel("RA")
    ax.set_ylabel("Dec")
fig.tight_layout()
         i = 0 deg
                              i = 45 deg
                                                  i = 90 deg
                                                                      i = 135 deg
                                                                                          i = 180 deg
  1.0
                       1.0
                                           1.0
                                                                1.0
                                                                                    1.0
  0.5
                       0.5
                                           0.5
                                                                0.5
                                                                                    0.5
ĕ 0.0
                     ĕ 0.0
                                         0.0
                                                             0.0
                                                                                  0.0
  -0.5
                      -0.5
                                           -0.5
                                                               -0.5
                                                                                    -0.5
  -1.0
        0.5
                            0.5
                                                    0.0
                                                                 1.0
                                                                     0.5
                                   -0.5 -1.0
                                             1.0
                                                0.5
                                                       -0.5 -1.0
                                                                            -0.5 -1.0
```

# What do the values of $\Omega$ mean?

 $\Omega$  or PAN defines the rotation of the orbit in the plane of the sky. Increasing PAN will rotate your orbit counterclockwise in the sky. Here are some examples:

```
[6]: sma = 1
    ecc, aop, plx, mtot = 0, 0, 1, 1
    inc = np.pi/4 # 45 degrees
    tau = 0.3

pans = np.radians([0, 30, 90, 150, 270])

all_eps = np.linspace(0, 365.25, 200)
    arc_eps = np.linspace(0, 75, 75)

fig = plt.figure(figsize=(15,4))
```

```
for i, pan in enumerate(pans):
    ax = fig.add\_subplot(1, 5, i+1)
    all_ras, all_decs, _ = calc_orbit(all_eps, sma, ecc, inc, aop, pan, tau, plx, mtot,_
→tau_ref_epoch=0)
     ax.plot(all_ras, all_decs, 'k--', alpha=0.5)
    arc_ras, arc_decs, _ = calc_orbit(arc_eps, sma, ecc, inc, aop, pan, tau, plx, mtot,__
→tau_ref_epoch=0)
     ax.plot(arc_ras, arc_decs, 'b-', linewidth=2)
    ax.plot(arc_ras[-1], arc_decs[-1], 'bo', markersize=7)
    ax.set\_title(r"\$\Omega = {0:d} deg".format(int(np.degrees(pan))))
    ax.set_aspect("equal")
    ax.set_xlim([1.1, -1.1])
    ax.set_ylim([-1.1, 1.1])
    ax.set_xlabel("RA")
    ax.set_ylabel("Dec")
fig.tight_layout()
                               \Omega = 29 \text{ deg}
                                                                          \Omega = 150 \text{ deg}
          Q = 0 \text{ deg}
                                                     \Omega = 90 \text{ deg}
                                                                                               0 = 270 \text{ deg}
   1.0
                        1.0
                                              1.0
                                                                   1.0
                                                                                         1.0
                        0.5
                                              0.5
ĕ 0.0
                      Dec
                        0.0
                                           Dec
                                             0.0
                                                                 Dec
                                                                   0.0
                                                                                       0.0
                                                                   -0.5
        0.5
                              0.5
                                     -0.5 -1.0
                                                1.0
```

# 2.4 Contributing to the Code

orbitize is under active development, and we've still got a lot to do! To get involved, check out our contributor guidelines, look over our issues list, and/or reach out to Sarah. We'd love to have you on our team!

Members of our team have collectively drafted this community agreement stating both our values and ground rules. In joining our team, we ask that you read and (optionally) suggest changes to this document.

# 2.5 Detailed API Documentation

# 2.5.1 Basis

Abstract base class for different basis sets. All new basis objects should inherit from this class. This class is meant to control how priors are assigned to various basis sets and how conversions are made from the basis sets to the standard keplarian set.

Author: Tirth, 2021

#### set\_default\_mass\_priors(priors\_arr, labels\_arr)

Adds the necessary priors for the stellar and/or companion masses.

#### **Parameters**

- **priors\_arr** (*list of orbitize.priors.Prior objects*) holds the prior objects for each parameter to be fitted (updated here)
- labels\_arr (list of strings) holds the name of all the parameters to be fitted (updated here)

# set\_hip\_iad\_priors(priors\_arr, labels\_arr)

Adds the necessary priors relevant to the hipparcos data to 'priors\_arr' and updates 'labels\_arr' with the priors' corresponding labels.

#### **Parameters**

- **priors\_arr** (*list of orbitize.priors.Prior objects*) holds the prior objects for each parameter to be fitted (updated here)
- labels\_arr (list of strings) holds the name of all the parameters to be fitted (updated here)

# set\_rv\_priors(priors\_arr, labels\_arr)

Adds the necessary priors if radial velocity data is supplied to 'priors\_arr' and updates 'labels\_arr' with the priors' corresponding labels. This function assumes that 'rv' data has been supplied and a secondary mass is being fitted for.

# **Parameters**

- **priors\_arr** (*list of orbitize.priors.Prior objects*) holds the prior objects for each parameter to be fitted (updated here)
- labels\_arr (list of strings) holds the name of all the parameters to be fitted (updated here)

# verify\_params()

Displays warnings about the 'fit\_secondary\_mass' and 'rv' parameters for all basis sets. Can be overriden by any basis class depending on the necessary parameters that need to be checked.

```
class orbitize.basis.Period(stellar_or_system_mass, mass_err, plx, plx_err, num_secondary_bodies, fit_secondary_mass, angle_upperlim=6.283185307179586, hipparcos IAD=None, rv=False, rv instruments=None)
```

Modification of the standard basis, swapping our sma for period: (per, ecc, inc, aop, pan, tau).

#### **Parameters**

- **stellar\_or\_system\_mass** (*float*) mass of the primary star (if fitting for dynamical masses of both components) or total system mass (if fitting using relative astrometry only) [M\_sol]
- mass\_err (float) uncertainty on 'stellar\_or\_system\_mass', in M\_sol
- plx (float) mean parallax of the system, in mas
- plx\_err (float) uncertainty on 'plx', in mas
- num\_secondary\_bodies (int) number of secondary bodies in the system, should be at least 1
- **fit\_secondary\_mass** (*bool*) if True, include the dynamical mass of orbitting body as fitted parameter, if False, 'stellar\_or\_system\_mass' is taken to be total mass

- **angle\_upperlim** (*float*) either pi or 2pi, to restrict the prior range for 'pan' parameter (default: 2pi)
- hipparcos\_IAD (orbitize.HipparcosLogProb object) if not 'None', then add relevant priors to this data (default: None)
- **rv** (*bool*) if True, then there is radial velocity data and assign radial velocity priors, if False, then there is no radial velocity data and radial velocity priors are not assigned (default: False)
- rv\_instruments (np. array) array of unique rv instruments from the originally supplied data (default: None)

#### construct\_priors()

Generates the parameter label array and initializes the corresponding priors for each parameter that's to be sampled. For the standard basis, the parameters common to each companion are: per, ecc, inc, aop, pan, tau. Parallax, hipparcos (optional), rv (optional), and mass priors are added at the end.

#### Returns

list: list of strings (labels) that indicate the names of each parameter to sample

list: list of orbitize.priors.Prior objects that indicate the prior distribution of each label

# Return type

tuple

# to\_period\_basis(param\_arr)

Convert parameter array from standard basis to period basis by swapping out the semi-major axis parameter to period for each companion. This function is used primarily for testing purposes.

#### **Parameters**

**param\_arr** (np.array of float) – RxM array of fitting parameters in the standard basis, where R is the number of parameters being fit, and M is the number of orbits. If M=1 (for MCMC), this can be a 1D array.

#### Returns

### modifies 'param\_arr' to contain the period for each companion

in each orbit rather than semi-major axis. Shape of 'param\_arr' remains the same.

#### Return type

np.array of float

# to\_standard\_basis(param\_arr)

Convert parameter array from period basis to standard basis by swapping out the period parameter to semimajor axis for each companion.

# **Parameters**

**param\_arr** (np.array of float) – RxM array of fitting parameters in the period basis, where R is the number of parameters being fit, and M is the number of orbits. If M=1 (for MCMC), this can be a 1D array.

#### Returns

### modifies 'param\_arr' to contain the semi-major axis for each companion

in each orbit rather than period. Shape of 'param\_arr' remains the same.

#### Return type

np.array of float

Modification of the standard basis, swapping our sma for period and additionally sampling in the stellar radial velocity semi-amplitude: (per, ecc, inc, aop, pan, tau, K).

**Note:** Ideally, 'fit\_secondary\_mass' is true and rv data is supplied.

#### **Parameters**

- **stellar\_or\_system\_mass** (*float*) mass of the primary star (if fitting for dynamical masses of both components) or total system mass (if fitting using relative astrometry only) [M\_sol]
- mass\_err (float) uncertainty on 'stellar\_or\_system\_mass', in M\_sol
- plx (float) mean parallax of the system, in mas
- plx\_err (float) uncertainty on 'plx', in mas
- num\_secondary\_bodies (int) number of secondary bodies in the system, should be at least 1
- **fit\_secondary\_mass** (*bool*) if True, include the dynamical mass of orbitting body as fitted parameter, if False, 'stellar\_or\_system\_mass' is taken to be total mass
- angle\_upperlim (float) either pi or 2pi, to restrict the prior range for 'pan' parameter (default: 2\*pi)
- hipparcos\_IAD (orbitize.HipparcosLogProb object) if not 'None', then add relevant priors to this data (default: None)
- **rv** (*bool*) if True, then there is radial velocity data and assign radial velocity priors, if False, then there is no radial velocity data and radial velocity priors are not assigned (default: False)
- **rv\_instruments** (*np.array*) array of unique rv instruments from the originally supplied data (default: None)

# $compute\_companion\_mass(period, ecc, inc, semi\_amp, m0)$

Computes a single companion's mass given period, eccentricity, inclination, stellar rv semi-amplitude, and stellar mass. Uses scipy.fsolve to compute the masses numerically.

#### **Parameters**

- **period** (*np.array of float*) the period values for each orbit for a single companion (can be float)
- **ecc** (*np.array of float*) the eccentricity values for each orbit for a single companion (can be float)
- inc (np.array of float) the inclination values for each orbit for a single companion (can be float)
- **semi\_amp** (*np.array of float*) the stellar rv-semi amp values for each orbit (can be float)
- m0 (np.array of float) the stellar mass for each orbit (can be float)

#### Returns

the companion mass values for each orbit (can also just be a single float)

#### Return type

np.array of float

#### compute\_companion\_sma( $period, m0, m_n$ )

Computes a single companion's semi-major axis using Kepler's Third Law for each orbit.

#### **Parameters**

- **period** (*np.array of float*) the period values for each orbit for a single companion (can be float)
- m0 (np.array of float) the stellar mass for each orbit (can be float)
- m\_n (np.array of float) the companion mass for each orbit (can be float)

#### Returns

the semi-major axis values for each orbit

# Return type

np.array of float

#### construct\_priors()

Generates the parameter label array and initializes the corresponding priors for each parameter that's to be sampled. For the semi-amp basis, the parameters common to each companion are: per, ecc, inc, aop, pan, tau, K (stellar rv semi-amplitude). Parallax, hipparcos (optional), rv (optional), and mass priors are added at the end.

The mass parameter will always be m0.

#### Returns

list: list of strings (labels) that indicate the names of each parameter to sample

list: list of orbitize.priors.Prior objects that indicate the prior distribution of each label

# Return type

tuple

# func(x, lhs, m0)

Define function for scipy.fsolve to use when computing companion mass.

#### **Parameters**

- **x** (*float*) the companion mass to be calculated (Msol)
- **lhs** (*float*) the left hand side of the rv semi-amplitude equation (Msol^(1/3))
- m0 (float) the stellar mass (Msol)

#### Returns

the difference between the rhs and lhs of the rv semi-amplitude equation, 'x' is a good companion mass when this difference is very close to zero

# Return type

float

# to\_semi\_amp\_basis(param\_arr)

Convert parameter array from standard basis to semi-amp basis by swapping out the semi-major axis parameter to period for each companion and computing the stellar rv semi-amplitudes for each companion.

# **Parameters**

**param\_arr** (np.array of float) – RxM array of fitting parameters in the period basis, where R is the number of parameters being fit, and M is the number of orbits. If M=1 (for MCMC), this can be a 1D array.

#### Returns

#### modifies 'param\_arr' to contain the semi-major axis for each companion

in each orbit rather than period, appends stellar rv semi-amplitude parameters, and removes companion masses

#### Return type

np.array of float

#### to\_standard\_basis(param arr)

Convert parameter array from semi-amp basis to standard basis by swapping out the period parameter to semi-major axis for each companion and computing the masses of each companion.

#### **Parameters**

**param\_arr** (np. array of float) – RxM array of fitting parameters in the period basis, where R is the number of parameters being fit, and M is the number of orbits. If M=1 (for MCMC), this can be a 1D array.

#### Returns

### modifies 'param\_arr' to contain the semi-major axis for each companion

in each orbit rather than period, removes stellar rv semi-amplitude parameters for each companion, and appends the companion masses to 'param\_arr'

#### Return type

np.array of float

#### verify\_params()

Additionally warns that this basis will sample stellar mass rather than sample mass regardless of whether 'fit secondary mass' is True or not.

class orbitize.basis.Standard(stellar\_or\_system\_mass, mass\_err, plx, plx\_err, num\_secondary\_bodies, fit\_secondary\_mass, angle\_upperlim=6.283185307179586, hipparcos\_IAD=None, rv=False, rv\_instruments=None)

Standard basis set based upon the 6 standard Keplarian elements: (sma, ecc, inc, aop, pan, tau).

- **stellar\_or\_system\_mass** (*float*) mass of the primary star (if fitting for dynamical masses of both components) or total system mass (if fitting using relative astrometry only) [M\_sol]
- mass\_err (float) uncertainty on 'stellar\_or\_system\_mass', in M\_sol
- plx (float) mean parallax of the system, in mas
- plx\_err (float) uncertainty on 'plx', in mas
- num\_secondary\_bodies (int) number of secondary bodies in the system, should be at least 1
- **fit\_secondary\_mass** (*bool*) if True, include the dynamical mass of orbitting body as fitted parameter, if False, 'stellar\_or\_system\_mass' is taken to be total mass
- angle\_upperlim (float) either pi or 2pi, to restrict the prior range for 'pan' parameter (default: 2pi)
- hipparcos\_IAD (orbitize.HipparcosLogProb object) if not 'None', then add relevant priors to this data (default: None)
- **rv** (*bool*) if True, then there is radial velocity data and assign radial velocity priors, if False, then there is no radial velocity data and radial velocity priors are not assigned (default: False)
- **rv\_instruments** (*np.array*) array of unique rv instruments from the originally supplied data (default: None)

#### construct\_priors()

Generates the parameter label array and initializes the corresponding priors for each parameter that's to be sampled. For the standard basis, the parameters common to each companion are: sma, ecc, inc, aop, pan, tau. Parallax, hipparcos (optional), rv (optional), and mass priors are added at the end.

#### Returns

list: list of strings (labels) that indicate the names of each parameter to sample

list: list of orbitize.priors.Prior objects that indicate the prior distribution of each label

# Return type

tuple

# to\_standard\_basis(param\_arr)

For standard basis, no conversion needs to be made.

### **Parameters**

**param\_arr** (np.array of float) – RxM array of fitting parameters in the standard basis, where R is the number of parameters being fit, and M is the number of orbits. If M=1 (for MCMC), this can be a 1d array.

#### Returns

param\_arr without any modification

# Return type

np.array of float

```
class orbitize.basis.XYZ(stellar_or_system_mass, mass_err, plx, plx_err, num_secondary_bodies, fit_secondary_mass, data_table, best_epoch_idx, epochs, angle_upperlim=6.283185307179586, hipparcos_IAD=None, rv=False, rv_instruments=None)
```

Defines an orbit using the companion's position and velocity components in XYZ space (x, y, z, xdot, ydot, zdot). The conversion algorithms used for this basis are defined in the following paper: http://www.dept.aoe.vt.edu/~lutze/AOE4134/9OrbitInSpace.pdf

Note: Does not have support with sep,pa data yet.

**Note:** Does not work for all multi-body data.

- **stellar\_or\_system\_mass** (*float*) mass of the primary star (if fitting for dynamical masses of both components) or total system mass (if fitting using relative astrometry only) [M\_sol]
- mass\_err (float) uncertainty on 'stellar\_or\_system\_mass', in M\_sol
- plx (float) mean parallax of the system, in mas
- plx\_err (float) uncertainty on 'plx', in mas
- num\_secondary\_bodies (int) number of secondary bodies in the system, should be at least 1
- **fit\_secondary\_mass** (*bool*) if True, include the dynamical mass of orbitting body as fitted parameter, if False, 'stellar\_or\_system\_mass' is taken to be total mass

- input\_table (astropy.table.Table) output from 'orbitize.read\_input.read\_file()'
- **best\_epoch\_idx** (list) indices of the epochs corresponding to the smallest uncertainties
- **epochs** (list) all of the astrometric epochs from 'input\_table'
- angle\_upperlim (float) either pi or 2pi, to restrict the prior range for 'pan' parameter (default: 2\*pi)
- hipparcos\_IAD (orbitize.HipparcosLogProb object) if not 'None', then add relevant priors to this data (default: None)
- **rv** (*bool*) if True, then there is radial velocity data and assign radial velocity priors, if False, then there is no radial velocity data and radial velocity priors are not assigned (default: False)
- rv\_instruments (np. array) array of unique rv instruments from the originally supplied data (default: None)

# Author: Rodrigo

# construct\_priors()

Generates the parameter label array and initializes the corresponding priors for each parameter that's to be sampled. For the xyz basis, the parameters common to each companion are: x, y, z, xdot, ydot, zdot. Parallax, hipparcos (optional), rv (optional), and mass priors are added at the end.

The xyz basis describes the position and velocity vectors with reference to the local coordinate system (the origin of the system is star).

#### Returns

list: list of strings (labels) that indicate the names of each parameter to sample

list: list of orbitize.priors.Prior objects that indicate the prior distribution of each label

# Return type

tuple

standard\_to\_xyz(epoch, elems, tau\_ref\_epoch=58849, tolerance=1e-09, max\_iter=100)

Converts array of orbital elements from the regular base of Keplerian orbits to positions and velocities in xyz Uses code from orbitize.kepler

# **Parameters**

- epoch (float) Date in MJD of observation to calculate time of periastron passage (tau).
- **elems** (*np.array of floats*) Orbital elements (sma, ecc, inc, aop, pan, tau, plx, mtot). If more than 1 set of parameters is passed, the first dimension must be the number of orbital parameter sets, and the second the orbital elements.

#### Returns

Orbital elements in xyz (x-coordinate [au], y-coordinate [au], z-coordinate [au], velocity in x [km/s], velocity in y [km/s], velocity in z [km/s], parallax [mas], total mass of the two-body orbit

 $(M_* + M_planet)$  [Solar masses])

### Return type

np.array

# to\_standard\_basis(param\_arr)

Makes a call to 'xyz\_to\_standard' to convert each companion's xyz parameters to the standard parameters an returns the updated array for conversion.

#### **Parameters**

**param\_arr** (np.array of float) – RxM array of fitting parameters in the period basis, where R is the number of parameters being fit, and M is the number of orbits. If M=1 (for MCMC), this can be a 1D array.

#### Returns

Orbital elements in the standard basis for all companions.

#### Return type

np.array

#### to\_xyz\_basis(param\_arr)

Makes a call to 'standard\_to\_xyz' to convert each companion's standard keplerian parameters to the xyz parameters an returns the updated array for conversion.

#### **Parameters**

**param\_arr** (np.array of float) – RxM array of fitting parameters in the period basis, where R is the number of parameters being fit, and M is the number of orbits. If M=1 (for MCMC), this can be a 1D array.

#### Returns

Orbital elements in the xyz for all companions.

#### Return type

np.array

# verify\_params()

For now, additionally throws exceptions if data is supplied in sep/pa or if the best epoch for each body is one of the last two (which would inevtably mess up how the priors are imposed).

# xyz\_to\_standard(epoch, elems, tau\_ref\_epoch=58849)

Converts array of orbital elements in terms of position and velocity in xyz to the standard basis.

#### **Parameters**

- epoch (float) Date in MJD of observation to calculate time of periastron passage (tau).
- **elems** (*np.array of floats*) Orbital elements in xyz basis (x-coordinate [au], y-coordinate [au], z-coordinate [au], velocity in x [km/s], velocity in y [km/s], velocity in z [km/s], parallax [mas], total mass of the two-body orbit (M\_\* + M\_planet) [Solar masses]). If more than 1 set of parameters is passed, the first dimension must be the number of orbital parameter sets, and the second the orbital elements.

#### Returns

#### Orbital elements in the standard basis

(sma, ecc, inc, aop, pan, tau, plx, mtot)

# Return type

np.array

orbitize.basis.switch\_tau\_epoch(old\_tau, old\_epoch, new\_epoch, period)

Convert tau to another tau that uses a different referench epoch

- old\_tau (float or np.array) old tau to convert
- old\_epoch (float or np.array) old reference epoch (days, typically MJD)
- new\_epoch (float or np.array) new reference epoch (days, same system as old\_epoch)

• **period** (*float or np.array*) – orbital period (years)

#### Returns

new taus

# Return type

float or np.array

orbitize.basis.tau\_to\_manom(date, sma, mtot, tau, tau\_ref\_epoch)

Gets the mean anomlay. Wrapper for kepler.tau to manom, kept here for backwards compatibility.

#### **Parameters**

- date (float or np.array) MJD
- sma (float) semi major axis (AU)
- **mtot** (*float*) total mass (M\_sun)
- tau (float) epoch of periastron, in units of the orbital period
- tau\_ref\_epoch (float) reference epoch for tau

#### Returns

mean anomaly on that date [0, 2pi)

#### Return type

float or np.array

orbitize.basis.tau\_to\_tp(tau, ref\_epoch, period, after\_date=None)

Convert tau (epoch of periastron in fractional orbital period after ref epoch) to t\_p (date in days, usually MJD, but works with whatever system ref\_epoch is given in)

# **Parameters**

- tau (float or np.array) value of tau to convert
- ref\_epoch (float or np.array) date (in days, typically MJD) that tau is defined relative to
- period (float or np.array) period (in years) that tau is noralized with
- after\_date (float or np.array) tp will be the first periastron after this date. If None, use ref\_epoch.

#### Returns

corresponding  $t_p$  of the taus

### **Return type**

float or np.array

orbitize.basis.tp\_to\_tau(tp, ref\_epoch, period)

Convert t\_p to tau

#### **Parameters**

- tp (float or np.array) value to t\_p to convert (days, typically MJD)
- **ref\_epoch** (*float or np.array*) reference epoch (in days) that tau is defined from. Same system as tp (e.g., MJD)
- **period** (*float or np.array*) period (in years) that tau is defined by

#### Returns

corresponding taus

#### Return type

float or np.array

#### **2.5.2** Driver

Runs through orbitize methods in a standardized way.

#### **Parameters**

- **input\_data** Either a relative path to data file or astropy.table.Table object in the orbitize format. See orbitize.read\_input
- **sampler\_str** (*str*) algorithm to use for orbit computation. "MCMC" for Markov Chain Monte Carlo, "OFTI" for Orbits for the Impatient
- num\_secondary\_bodies (int) number of secondary bodies in the system. Should be at least 1.
- **stellar\_or\_system\_mass** (*float*) mass of the primary star (if fitting for dynamical masses of both components) or total system mass (if fitting using relative astrometry only) [M\_sol]
- plx (float) mean parallax of the system [mas]
- mass\_err (float, optional) uncertainty on stellar\_or\_system\_mass [M\_sol]
- plx\_err (float, optional) uncertainty on plx [mas]
- **Inlike** (*str*, *optional*) name of function in orbitize.lnlike that will be used to compute likelihood. (default="chi2\_lnlike")
- chi2\_type (str, optional) either "standard", or "log"
- system\_kwargs (dict, optional) restrict\_angle\_ranges, tau\_ref\_epoch, fit\_secondary\_mass, hipparcos\_IAD, gaia, use\_rebound, fitting\_basis for orbitize.system.System.
- mcmc\_kwargs (dict, optional) num\_temps, num\_walkers, and num\_threads kwargs for orbitize.sampler.MCMC

Written: Sarah Blunt, 2018

### 2.5.3 Gaia API Module

**class** orbitize.gaia.**GaiaLogProb**(gaia\_num, hiplogprob, dr='dr2', query=True, gaia\_data=None)

Class to compute the log probability of an orbit with respect to a single astrometric position point from Gaia. Uses astroquery to look up Gaia astrometric data, and computes log-likelihood. To be used in conjunction with orbitize.hipparcos.HipLogProb; see documentation for that object for more detail.

Follows Nielsen+ 2020 (studying the orbit of beta Pic b). Note that this class currently only fits for the position of the star in the Gaia epoch, not the star's proper motion.

**Note:** In orbitize, it is possible to perform a fit to just the Hipparcos IAD, but not to just the Gaia astrometric data.

#### **Parameters**

- gaia\_num (int) the Gaia source ID of the object you're fitting. Note that the dr2 and edr3 source IDs are not necessarily the same.
- **hiplogprob** (*orbitize.hipparcos.HipLogProb*) object containing all info relevant to Hipparcos IAD fitting
- **dr** (*str*) either 'dr2' or 'edr3'
- **query** (*bool*) if True, queries the Gaia database for astrometry of the target (requires an internet connection). If False, uses user-input astrometric values (runs without internet).
- **gaia\_data** (*dict*) see *query* keyword above. If *query* set to False, then user must supply a dictionary of Gaia astometry in the following form:

Written: Sarah Blunt, 2021

compute\_lnlike(raoff\_model, deoff\_model, samples, param\_idx)

Computes the log likelihood of an orbit model with respect to a single Gaia astrometric point. This is added to the likelihoods calculated with respect to other data types in sampler.\_logl().

#### **Parameters**

- raoff\_model (np.array of float) 2xM primary RA offsets from the barycenter incurred from orbital motion of companions (i.e. not from parallactic motion), where M is the number of orbits being tested, and raoff\_model[0,:] are position predictions at the Hipparcos epoch, and raoff\_model[1,:] are position predictions at the Gaia epoch
- **deoff\_model** (*np.array of float*) 2xM primary decl offsets from the barycenter incurred from orbital motion of companions (i.e. not from parallactic motion), where M is the number of orbits being tested, and deoff\_model[0,:] are position predictions at the Hipparcos epoch, and deoff\_model[1,:] are position predictions at the Gaia epoch
- **samples** (*np.array of float*) R-dimensional array of fitting parameters, where R is the number of parameters being fit. Must be in the same order documented in System.
- **param\_idx** a dictionary matching fitting parameter labels to their indices in an array of fitting parameters (generally set to System.basis.param idx).

#### Returns

# array of length M, where M is the number of input

orbits, representing the log likelihood of each orbit with respect to the Gaia position measurement.

#### Return type

np.array of float

# 2.5.4 Hipparcos API Module

**class** orbitize.hipparcos.**HipparcosLogProb**(path\_to\_iad\_file, hip\_num, num\_secondary\_bodies, alphadec0\_epoch=1991.25, renormalize\_errors=False)

Class to compute the log probability of an orbit with respect to the Hipparcos Intermediate Astrometric Data (IAD). If using a DVD file, queries Vizier for all metadata relevant to the IAD, and reads in the IAD themselves from a specified location. Follows Nielsen+ 2020 (studying the orbit of beta Pic b).

Fitting the Hipparcos IAD requires fitting for the following five parameters. They are added to the vector of fitting parameters in system.py, but are described here for completeness. See Nielsen+ 2020 for more detail.

- alpha0: RA offset from the reported Hipparcos position at a particular epoch (usually 1991.25) [mas]
- delta0: Dec offset from the reported Hipparcos position at a particular epoch (usually 1991.25) [mas]
- pm\_ra: RA proper motion [mas/yr]
- pm\_dec: Dec proper motion [mas/yr]
- · plx: parallax [mas]

**Note:** In orbitize, it is possible to perform a fit to just the Hipparcos IAD, but not to just the Gaia astrometric data.

#### **Parameters**

- path\_to\_iad\_file (str) location of IAD file to be used in your fit. See the Hipparcos tutorial for a walkthrough of how to download these files.
- **hip\_num** (*str*) Hipparcos ID of star. Available on Simbad. Should have zeros in the prefix if number is <100,000. (i.e. 27321 should be passed in as '027321').
- num\_secondary\_bodies (int) number of companions in the system
- **alphadec0\_epoch** (*float*) epoch (in decimal year) that the fitting parameters alpha0 and delta0 are defined relative to (see above).
- **renormalize\_errors** (*bool*) if True, normalize the scan errors to get chisq\_red = 1, following Nielsen+ 2020 (eq 10). In general, this should be False, but it's helpful for testing. Check out *orbitize.hipparcos.nielsen\_iad\_refitting\_test()* for an example using this renormalization.

Written: Sarah Blunt & Rob de Rosa, 2021

**compute\_lnlike**(raoff\_model, deoff\_model, samples, param\_idx)

Computes the log likelihood of an orbit model with respect to the Hipparcos IAD. This is added to the likelihoods calculated with respect to other data types in sampler.\_logl().

- raoff\_model (np.array of float) M-dimensional array of primary RA offsets from the barycenter incurred from orbital motion of companions (i.e. not from parallactic motion), where M is the number of epochs of IAD scan data.
- **deoff\_model** (*np.array of float*) M-dimensional array of primary RA offsets from the barycenter incurred from orbital motion of companions (i.e. not from parallactic motion), where M is the number of epochs of IAD scan data.

- **samples** (*np.array of float*) R-dimensional array of fitting parameters, where R is the number of parameters being fit. Must be in the same order documented in System.
- **param\_idx** a dictionary matching fitting parameter labels to their indices in an array of fitting parameters (generally set to System.basis.param\_idx).

### Returns

### array of length M, where M is the number of input

orbits, representing the log likelihood of each orbit with respect to the Hipparcos IAD.

#### **Return type**

np.array of float

Reproduce the IAD refitting test from Nielsen+ 2020 (end of Section 3.1). The default MCMC parameters are what you'd want to run before using the IAD for a new system. This fit uses 100 walkers.

#### **Parameters**

- **iad\_loc** (*str*) path to the IAD file.
- **hip\_num** (*str*) Hipparcos ID of star. Available on Simbad. Should have zeros in the prefix if number is <100,000. (i.e. 27321 should be passed in as '027321').
- **saveplot** (*str*) what to save the summary plot as. If None, don't make a plot
- **burn\_steps** (*int*) number of MCMC burn-in steps to run.
- mcmc\_steps (int) number of MCMC production steps to run.

#### Returns

numpy.array of float: n\_steps x 5 array of posterior samples

# **orbitize.hipparcos.HipparcosLogProb: the object storing relevant** metadata for the performed Hipparcos IAD fit

#### Return type

tuple

# 2.5.5 Kepler Solver

This module solves for the orbit of the planet given Keplerian parameters.

```
orbitize.kepler.calc_orbit(epochs, sma, ecc, inc, aop, pan, tau, plx, mtot, mass_for_Kamp=None, tau_ref_epoch=58849, tolerance=1e-09, max_iter=100, use_c=True, use_gpu=False)
```

Returns the separation and radial velocity of the body given array of orbital parameters (size  $n_{orbs}$ ) at given epochs (array of size  $n_{dates}$ )

Based on orbit solvers from James Graham and Rob De Rosa. Adapted by Jason Wang and Henry Ngo.

- epochs (np. array) MJD times for which we want the positions of the planet
- sma (np.array) semi-major axis of orbit [au]
- ecc (np.array) eccentricity of the orbit [0,1]

- **inc** (*np.array*) inclination [radians]
- aop (np. array) argument of periastron [radians]
- pan (np.array) longitude of the ascending node [radians]
- tau (np. array) epoch of periastron passage in fraction of orbital period past MJD=0 [0,1]
- plx (np.array) parallax [mas]
- **mtot** (*np.array*) total mass of the two-body orbit (M\_\* + M\_planet) [Solar masses]
- mass\_for\_Kamp (np.array, optional) mass of the body that causes the RV signal. For example, if you want to return the stellar RV, this is the planet mass. If you want to return the planetary RV, this is the stellar mass. [Solar masses]. For planet mass ~ 0, mass\_for\_Kamp ~ M\_tot, and function returns planetary RV (default).
- tau\_ref\_epoch (float, optional) reference date that tau is defined with respect to (i.e., tau=0)
- **tolerance** (*float*, *optional*) absolute tolerance of iterative computation. Defaults to 1e-9.
- max\_iter (int, optional) maximum number of iterations before switching. Defaults to 100.
- $use\_c \ (bool, optional)$  Use the C solver if configured. Defaults to True
- use\_gpu (bool, optional) Use the GPU solver if configured. Defaults to False

#### Returns

raoff (np.array): array-like ( $n_{array}$ ): array-like ( $n_{array}$ ) of RA offsets between the bodies (origin is at the other body) [mas]

deoff (np.array): array-like (n\_dates x n\_orbs) of Dec offsets between the bodies [mas]

vz (np.array): array-like (n\_dates x n\_orbs) of radial velocity of one of the bodies (see mass\_for\_Kamp description) [km/s]

#### Return type

3-tuple

Written: Jason Wang, Henry Ngo, 2018

orbitize.kepler.tau\_to\_manom(date, sma, mtot, tau, tau ref epoch)

Gets the mean anomlay

#### **Parameters**

- date (float or np.array) MJD
- sma (float) semi major axis (AU)
- **mtot** (*float*) total mass (M\_sun)
- tau (float) epoch of periastron, in units of the orbital period
- tau\_ref\_epoch (float) reference epoch for tau

#### Returns

mean anomaly on that date [0, 2pi)

# Return type

float or np.array

# 2.5.6 Log(Likelihood)

orbitize.lnlike.chi2\_lnlike(data, errors, corrs, model, jitter, seppa\_indices, chi2\_type='standard')

Compute Log of the chi2 Likelihood

### Args:

data (np.array): Nobsx2 array of data, where data[:,0] = sep/RA/RV for every epoch, and data[:,1] = corresponding pa/DEC/np.nan.

errors (np.array): Nobsx2 array of errors for each data point. Same format as data.

#### corrs (np.array): Nobs array of Pearson correlation coeffs

between the two quantities. If there is none, can be None.

model (np.array): Nobsx2xM array of model predictions, where M is the number of orbits being compared against the data. If M is 1, model can be 2 dimensional. jitter (np.array): Nobsx2xM array of jitter values to add to errors.

Elements of array should be 0 for for all data other than stellar rvs.

# $seppa\_indices\ (list) \hbox{:}\ list\ of\ epoch\ numbers\ whose\ observations\ are$

given in sep/PA. This list is located in System.seppa.

chi2\_type (string): the format of chi2 to use is either 'standard' or 'log'

#### **Returns:**

np.array: Nobsx2xM array of chi-squared values.

**Note:** (1) **Example**: We have 8 epochs of data for a system. OFTI returns an array of 10,000 sets of orbital parameters. The model input for this function should be an array of dimension  $8 \times 2 \times 10,000$ .

(2) Chi2\_log: redefining chi-squared in log scale may give a more stable optimization. This works on separation and position angle data (seppa) not right ascension and declination (radec) data, but it is possible to convert between the two within Orbitize! using the function 'orbitize.system'radec2seppa' (see docuemntation). This implementation defines sep chi-squared in log scale, and defines pa chi-sq using complex phase representation. log sep chisq = (log sep - log sep\_true)^2 / (sep\_sigma / sep\_true)^2 pa chisq = 2 \* (1 - cos(pa-pa\_true))/pa\_sigma^2

i

orbitize.lnlike.chi2\_norm\_term(errors, corrs)

Return only the normalization term of the Gaussian likelihood:

$$-log(\sqrt(2\pi * error^2))$$

or

$$-0.5*(log(det(C)) + N*log(2\pi))$$

- errors (np.array) Nobsx2 array of errors for each data point. Same format as data.
- **corrs** (*np.array*) Nobs array of Pearson correlation coeffs between the two quantities. If there is none, can be None.

#### Returns

sum of the normalization terms

# Return type

float

# 2.5.7 N-body Backend

orbitize.nbody.calc\_orbit(epochs, sma, ecc, inc, aop, pan, tau, plx, mtot, tau\_ref\_epoch=58849, m\_pl=None, output\_star=False)

Solves for position for a set of input orbital elements using rebound.

#### **Parameters**

- epochs (np. array) MJD times for which we want the positions of the planet
- sma (np.array) semi-major axis of orbit [au]
- ecc (np.array) eccentricity of the orbit [0,1]
- **inc** (*np.array*) inclination [radians]
- aop (np. array) argument of periastron [radians]
- pan (np.array) longitude of the ascending node [radians]
- tau (np. array) epoch of periastron passage in fraction of orbital period past MJD=0 [0,1]
- plx (np.array) parallax [mas]
- **mtot** (*np.array*) total mass of the two-body orbit (M\_\* + M\_planet) [Solar masses]
- tau\_ref\_epoch (float, optional) reference date that tau is defined with respect to
- **m\_pl** (*np.array*, *optional*) mass of the planets[units]
- **output\_star** (*bool*) if True, also return the position of the star relative to the barycenter.

#### Returns

```
raoff (np.array): array-like (n_dates x n_orbs) of RA offsets between the bodies (origin is at the other body) [mas]
```

**deoff (np.array): array-like (n\_dates x n\_orbs) of Dec offsets between** the bodies [mas]

vz (np.array): array-like (n\_dates x n\_orbs) of radial velocity of one of the bodies (see *mass\_for\_Kamp* description) [km/s]

#### Return type

3-tuple

**Note:** return is in format [raoff[planet1, planet2,...,planetn], deoff[planet1, planet2,...,planetn], vz[planet1, planet2,...,planetn]

# 2.5.8 Plotting Methods

```
orbitize.plot.plot_corner(results, param_list=None, **corner_kwargs)
```

Make a corner plot of posterior on orbit fit from any sampler

#### **Parameters**

• param\_list (list of strings) – each entry is a name of a parameter to include. Valid strings:

```
sma1: semimajor axis
ecc1: eccentricity
inc1: inclination
aop1: argument of periastron
pan1: position angle of nodes
tau1: epoch of periastron passage, expressed as fraction of orbital.
→period
per1: period
K1: stellar radial velocity semi-amplitude
[repeat for 2, 3, 4, etc if multiple objects]
plx: parallax
pm_ra: RA proper motion
pm_dec: Dec proper motion
alpha0: primary offset from reported Hipparcos RA @ alphadec0_epoch_
→(generally 1991.25)
delta0: primary offset from reported Hipparcos Dec @ alphadec0_epoch_
\hookrightarrow (generally 1991.25)
gamma: rv offset
sigma: rv jitter
mi: mass of individual body i, for i = 0, 1, 2, ... (only if fit_
→secondary_mass == True)
mtot: total mass (only if fit_secondary_mass == False)
```

• \*\*corner\_kwargs – any remaining keyword args are sent to corner. See here.

Note: default axis labels used unless overwritten by user input.

# Returns

corner plot

# Return type

matplotlib.pyplot.Figure

**Note:** Example: Use param\_list = ['sma1,ecc1,inc1,sma2,ecc2,inc2'] to only plot posteriors for semimajor axis, eccentricity and inclination of the first two companions

```
Written: Henry Ngo, 2018
```

```
orbitize.plot.plot_orbits(results, object_to_plot=1, start_mjd=51544.0, num_orbits_to_plot=100, num_epochs_to_plot=100, square_plot=True, show_colorbar=True, cmap=<matplotlib.colors.LinearSegmentedColormap object>, sep_pa_color='lightgrey', sep_pa_end_year=2025.0, cbar_param='Epoch [year]', mod180=False, rv_time_series=False, plot_astrometry=True, plot_astrometry_insts=False, plot_errorbars=True, fig=None)
```

Plots one orbital period for a select number of fitted orbits for a given object, with line segments colored according to time

#### **Parameters**

- **object\_to\_plot** (*int*) which object to plot (default: 1)
- start\_mjd (float) MJD in which to start plotting orbits (default: 51544, the year 2000)
- num\_orbits\_to\_plot (int) number of orbits to plot (default: 100)
- num\_epochs\_to\_plot (int) number of points to plot per orbit (default: 100)
- **square\_plot** (*Boolean*) Aspect ratio is always equal, but if square\_plot is True (default), then the axes will be square, otherwise, white space padding is used
- **show\_colorbar** (*Boolean*) Displays colorbar to the right of the plot [True]
- **cmap** (*matplotlib.cm.ColorMap*) color map to use for making orbit tracks (default: modified Purples\_r)
- **sep\_pa\_color** (*string*) any valid matplotlib color string, used to set the color of the orbit tracks in the Sep/PA panels (default: 'lightgrey').
- **sep\_pa\_end\_year** (*float*) decimal year specifying when to stop plotting orbit tracks in the Sep/PA panels (default: 2025.0).
- **cbar\_param** (*string*) options are the following: 'Epoch [year]', 'sma1', 'ecc1', 'inc1', 'aop1', 'pan1', 'tau1', 'plx. Number can be switched out. Default is Epoch [year].
- mod180 (Bool) if True, PA will be plotted in range [180, 540]. Useful for plotting short arcs with PAs that cross 360 deg during observations (default: False)
- **rv\_time\_series** (*Boolean*) if fitting for secondary mass using MCMC for rv fitting and want to display time series, set to True.
- plot\_astrometry (Boolean) set to True by default. Plots the astrometric data.
- **plot\_astrometry\_insts** (*Boolean*) set to False by default. Plots the astrometric data by instruments.
- plot\_errorbars (Boolean) set to True by default. Plots error bars of measurements
- **fig** (matplotlib.pyplot.Figure) optionally include a predefined Figure object to plot the orbit on. Most users will not need this keyword.

# Returns

the orbit plot if input is valid, None otherwise

#### Return type

matplotlib.pyplot.Figure

(written): Henry Ngo, Sarah Blunt, 2018 Additions by Malena Rice, 2019

# 2.5.9 Priors

**class** orbitize.priors.**GaussianPrior**(*mu*, *sigma*, *no\_negatives=True*)

Gaussian prior.

$$log(p(x|\sigma,\mu)) \propto \frac{(x-\mu)}{\sigma}$$

- mu (float) mean of the distribution
- **sigma** (*float*) standard deviation of the distribution

• **no\_negatives** (*bool*) – if True, only positive values will be drawn from this prior, and the probability of negative values will be 0 (default:True).

(written) Sarah Blunt, 2018

# compute\_lnprob(element\_array)

Compute log(probability) of an array of numbers wrt a Gaussian distibution. Negative numbers return a probability of -inf.

#### **Parameters**

**element\_array** (*float* or np.array of float) – array of numbers. We want the probability of drawing each of these from the appropriate Gaussian distribution

#### **Returns**

array of log(probability) values, corresponding to the probability of drawing each of the numbers in the input *element\_array*.

#### **Return type**

numpy array of float

### draw\_samples(num\_samples)

Draw positive samples from a Gaussian distribution. Negative samples will not be returned.

#### **Parameters**

**num\_samples** (*float*) – the number of samples to generate

#### Returns

samples drawn from the appropriate Gaussian distribution. Array has length num samples.

#### Return type

numpy array of float

class orbitize.priors.KDEPrior(gaussian\_kde, total\_params, bounds=[], log\_scale\_arr=[])

Gaussian kernel density estimation (KDE) prior. This class is a wrapper for scipy.stats.gaussian\_kde.

#### **Parameters**

- gaussian\_kde (scipy.stats.gaussian\_kde) scipy KDE object containing the KDE defined by the user
- total\_params (float) number of parameters in the KDE
- **bounds** (array\_like of bool, optional) bounds for the KDE out of which the probreturned is -Inf
- bounds if True for a parameter the parameter is fit to the KDE in log-scale

Written: Jorge LLop-Sayson, Sarah Blunt (2021)

### compute\_lnprob(element\_array)

Compute log(probability) of an array of numbers wrt a the defined KDE. Negative numbers return a probability of -inf.

#### **Parameters**

**element\_array** (*float* or *np.array* of *float*) – array of numbers. We want the probability of drawing each of these from the KDE.

# Returns

array of log(probability) values, corresponding to the probability of drawing each of the numbers in the input *element\_array*.

#### Return type

numpy array of float

# draw\_samples(num\_samples)

Draw positive samples from the KDE. Negative samples will not be returned.

#### **Parameters**

**num\_samples** (*float*) – the number of samples to generate.

#### Returns

samples drawn from the KDE distribution. Array has length num\_samples.

#### Return type

numpy array of float

# increment\_param\_num()

Increment the index to evaluate the appropriate parameter.

# class orbitize.priors.LinearPrior(m, b)

Draw samples from the probability distribution:

$$p(x) \propto mx + b$$

where m is negative, b is positive, and the range is [0,-b/m].

#### **Parameters**

- m (float) slope of line. Must be negative.
- **b** (*float*) y intercept of line. Must be positive.

# draw\_samples(num\_samples)

Draw samples from a descending linear distribution.

#### **Parameters**

**num\_samples** (*float*) – the number of samples to generate

#### **Returns**

samples ranging from [0, -b/m) as floats.

#### Return type

np.array

# class orbitize.priors.LogUniformPrior(minval, maxval)

This is the probability distribution  $p(x) \propto 1/x$ 

The <u>\_\_init\_\_</u> method should take in a "min" and "max" value of the distribution, which correspond to the domain of the prior. (If this is not implemented, the prior has a singularity at 0 and infinite integrated probability).

#### **Parameters**

- **minval** (*float*) the lower bound of this distribution
- maxval (float) the upper bound of this distribution

#### compute\_lnprob(element array)

Compute the prior probability of each element given that its drawn from a Log-Uniofrm prior

# **Parameters**

element\_array (float or np.array of float) - array of paramters to compute the
prior probability of

#### **Returns**

array of prior probabilities

# Return type

np.array

### draw\_samples(num\_samples)

Draw samples from this 1/x distribution.

#### **Parameters**

**num\_samples** (*float*) – the number of samples to generate

#### Returns

samples ranging from [minval, maxval) as floats.

# Return type

np.array

# class orbitize.priors.NearestNDInterpPrior(interp\_fct, total\_params)

Nearest Neighbor interp. This class is a wrapper for scipy.interpolate.NearestNDInterpolator.

#### **Parameters**

- interp\_fct (scipy.interpolate.NearestNDInterpolator) scipy Interpolator object containing the NDInterpolator defined by the user
- total\_params (float) number of parameters

Written: Jorge LLop-Sayson (2021)

# compute\_lnprob(element\_array)

Compute log(probability) of an array of numbers wrt a the defined ND interpolator. Negative numbers return a probability of -inf.

#### **Parameters**

**element\_array** (*float or np.array of float*) – array of numbers. We want the probability of drawing each of these from the ND interpolator.

#### Returns

array of log(probability) values, corresponding to the probability of drawing each of the numbers in the input *element\_array*.

#### Return type

numpy array of float

#### draw\_samples(num\_samples)

Draw positive samples from the ND interpolator. Negative samples will not be returned.

#### **Parameters**

**num\_samples** (*float*) – the number of samples to generate.

#### Returns

samples drawn from the ND interpolator distribution. Array has length num samples.

#### **Return type**

numpy array of float

# increment\_param\_num()

Increment the index to evaluate the appropriate parameter.

### class orbitize.priors.Prior

Abstract base class for prior objects. All prior objects should inherit from this class.

Written: Sarah Blunt, 2018

# class orbitize.priors.SinPrior

This is the probability distribution  $p(x) \propto sin(x)$ 

The domain of this prior is [0,pi].

#### compute\_lnprob(element\_array)

Compute the prior probability of each element given that its drawn from a sine prior

#### **Parameters**

**element\_array** (*float or np.array of float*) — array of paramters to compute the prior probability of

#### Returns

array of prior probabilities

# Return type

np.array

# draw\_samples(num\_samples)

Draw samples from a Sine distribution.

#### **Parameters**

**num\_samples** (*float*) – the number of samples to generate

#### Returns

samples ranging from [0, pi) as floats.

#### Return type

np.array

# class orbitize.priors.UniformPrior(minval, maxval)

This is the probability distribution p(x) propto constant.

# **Parameters**

- minval (float) the lower bound of the uniform prior
- maxval (float) the upper bound of the uniform prior

# compute\_lnprob(element\_array)

Compute the prior probability of each element given that its drawn from this uniform prior

#### **Parameters**

element\_array (float or np.array of float) - array of paramters to compute the
prior probability of

# Returns

array of prior probabilities

# Return type

np.array

### draw\_samples(num\_samples)

Draw samples from this uniform distribution.

#### **Parameters**

**num\_samples** (*float*) – the number of samples to generate

#### **Returns**

samples ranging from [0, pi) as floats.

### Return type

np.array

# orbitize.priors.all\_Inpriors(params, priors)

Calculates log(prior probability) of a set of parameters and a list of priors

- params (np. array) size of N parameters
- priors (list) list of N prior objects corresponding to each parameter

#### Returns

prior probability of this set of parameters

# Return type

float

# 2.5.10 Read Input

Module to read user input from files and create standardized input for orbitize

```
orbitize.read_input.read_file(filename)
```

Reads data from any file for use in orbitize readable by astropy.io.ascii.read(), including csv format. See the astropy docs.

There are two ways to provide input data to orbitize.

The first way is to provide astrometric measurements, shown with the following example.

Example of an orbitize-readable .csv input file:

```
epoch,object,raoff,raoff_err,decoff,decoff_err,radec_corr,sep,sep_err,pa,pa_err,rv,

orv_err

1234,1,0.010,0.005,0.50,0.05,0.025,,,,,,

1235,1,,,,,1.0,0.005,89.0,0.1,,

1236,1,,,,,1.0,0.005,89.3,0.3,,

1237,0,,,,,,,,10,0.1
```

Each row must have epoch (in MJD=JD-2400000.5) and object. Objects are numbered with integers, where the primary/central object is 0. If you have, for example, one RV measurement of a star and three astrometric measurements of an orbiting planet, you should put 0 in the object column for the RV point, and 1 in the columns for the astrometric measurements.

Each line must also have at least one of the following sets of valid measurements:

- RA and DEC offsets [mas], or
- sep [mas] and PA [degrees East of NCP], or
- RV measurement [km/s]

**Note:** Columns with no data can be omitted (e.g. if only separation and PA are given, the raoff, deoff, and rv columns can be excluded).

If more than one valid set is given (e.g. RV measurement and astrometric measurement taken at the same epoch), read\_file() will generate a separate output row for each valid set.

For RA/Dec and Sep/PA, you can also specify a correlation term. This is useful when your error ellipse is tilted with respect to the RA/Dec or Sep/PA. The correlation term is the Pearson correlation coefficient (), which corresponds to the normalized off diagonal term of the covariance matrix. Let's define the covariance matrix as C = [[C\_11, C\_12], [C\_12, C\_22]]. Here C\_11 = quant1\_err^2 and C\_22 = quant2\_err^2 and C\_12 is the off diagonal term. Then = C\_12/(sqrt(C\_11)sqrt(C\_22)). Essentially it is the covariance normalized by the variance. As such, -1 1. You can specify either as radec\_corr or seppa\_corr. By definition, both are dimensionless, but one will correspond to RA/Dec and the other to Sep/PA. An example of real world data that

reports correlations is this GRAVITY paper where table 2 reports the correlation values and figure 4 shows how the error ellipses are tilted.

Alternatively, you can also supply a data file with the columns already corresponding to the orbitize format (see the example in description of what this method returns). This may be useful if you are wanting to use the output of the <code>write\_orbitize\_input</code> method.

**Note:** RV measurements of objects that are not the primary should be relative to the barycenter RV. For example, if the barycenter has a RV of 20 + -1 km/s, and you've measured an absolute RV for the secondary of 15 + -2 km/s, you should input an RV of -5.0 + -2.2 for object 1.

**Note:** When providing data with columns in the orbitize format, there should be no empty cells. As in the example below, when quant2 is not applicable, the cell should contain nan.

# **Parameters**

**filename** (str or astropy.table.Table) – Input filename or the actual table object

#### Returns

Table containing orbitize-readable input for given object. For the example input above:

epoch →type	object	quant1	quant1_err	quant2	quant2_err	quant12_corr	quant_
float64	int	float64	float64	float64	float64	float64	ш
⇔str5							
1234.0	1	0.01	0.005	0.5	0.05	0.025	ш
⊶radec							
1235.0	1	1.0	0.005	89.0	0.1	nan	ш
⊶seppa							
1236.0	1	1.0	0.005	89.3	0.3	nan	ш
⇔seppa							
1237.0	0	10.0	0.1	nan	nan	nan	ш

where quant\_type is one of "radec", "seppa", or "rv".

If quant\_type is "radec" or "seppa", the units of quant are mas and degrees, if quant\_type is "rv", the units of quant are km/s

### **Return type**

astropy. Table

Written: Henry Ngo, 2018

Updated: Vighnesh Nagpal, Jason Wang (2020-2021)

orbitize.read\_input.write\_orbitize\_input(table, output\_filename, file\_type='csv')

Writes orbitize-readable input as an ASCII file

# **Parameters**

• **table** (*astropy.Table*) – Table containing orbitize-readable input for given object, as generated by the read functions in this module.

- output\_filename (str) csv file to write to
- file\_type (str) Any valid write format for astropy.io.ascii. See the astropy docs. Defaults to csv.

(written) Henry Ngo, 2018

### **2.5.11 Results**

**class** orbitize.results.**Results**(*system=None*, *sampler\_name=None*, *post=None*, *lnlike=None*, *version\_number=None*, *curr\_pos=None*)

A class to store accepted orbital configurations from the sampler

#### **Parameters**

- **system** (orbitize.system.System) System object used to do the fit.
- **sampler\_name** (*string*) name of sampler class that generated these results (default: None).
- **post** (*np.array of float*) MxN array of orbital parameters (posterior output from orbit-fitting process), where M is the number of orbits generated, and N is the number of varying orbital parameters in the fit (default: None).
- **Inlike** (*np.array of float*) M array of log-likelihoods corresponding to the orbits described in post (default: None).
- **version\_number** (*str*) version of orbitize that produced these results.
- data (astropy.table.Table) output from orbitize.read\_input.read\_file()
- **curr\_pos** (*np.array of float*) for MCMC only. A multi-D array of the current walker positions that is used for restarting a MCMC sampler.

Written: Henry Ngo, Sarah Blunt, 2018

API Update: Sarah Blunt, 2021

add\_samples(orbital\_params, lnlikes, curr\_pos=None)

Add accepted orbits, their likelihoods, and the orbitize version number to the results

#### **Parameters**

- orbital\_params (np.array) add sets of orbital params (could be multiple) to results
- **Inlike** (*np.array*) add corresponding Inlike values to results
- **curr\_pos** (*np.array of float*) for MCMC only. A multi-D array of the current walker positions

Written: Henry Ngo, 2018

API Update: Sarah Blunt, 2021

load\_results(filename, append=False)

Populate the results. Results object with data from a datafile

- **filename** (*string*) filepath where data is saved
- **append** (*boolean*) if True, then new data is added to existing object. If False (default), new data overwrites existing object

See the save\_results() method in this module for information on how the data is structured.

Written: Henry Ngo, 2018

API Update: Sarah Blunt, 2021

plot\_corner(param\_list=None, \*\*corner\_kwargs)

Wrapper for orbitize.plot.plot corner

Wrapper for orbitize.plot.plot\_orbits

### print\_results()

Prints median and 68% credible intervals alongside fitting labels

#### save\_results(filename)

Save results. Results object to an hdf5 file

#### **Parameters**

**filename** (*string*) – filepath to save to

Save attributes from the results.Results object.

sampler\_name, tau\_ref\_epcoh, version\_number are attributes of the root group. post, lnlike, and parameter\_labels are datasets that are members of the root group.

Written: Henry Ngo, 2018

API Update: Sarah Blunt, 2021

# **2.5.12 Sampler**

MCMC sampler. Supports either parallel tempering or just regular MCMC. Parallel tempering will be run if num\_temps > 1 Parallel-Tempered MCMC Sampler uses ptemcee, a fork of the emcee Affine-infariant sampler Affine-Invariant Ensemble MCMC Sampler uses emcee.

**Warning:** may not work well for multi-modal distributions

- **system** (system.System) system.System object
- num\_temps (int) number of temperatures to run the sampler at. Parallel tempering will be used if num\_temps > 1 (default=20)
- **num\_walkers** (*int*) number of walkers at each temperature (default=1000)
- **num\_threads** (*int*) number of threads to use for parallelization (default=1)
- chi2\_type (str, optional) either "standard", or "log"

- like (str) name of likelihood function in lnlike.py
- **custom\_lnlike** (*func*) ability to include an addition custom likelihood function in the fit. The function looks like clnlikes = custon\_lnlike(params) where params is a RxM array of fitting parameters, where R is the number of orbital parameters (can be passed in system.compute\_model()), and M is the number of orbits we need model predictions for. It returns clnlikes which is an array of length M, or it can be a single float if M = 1.
- **prev\_result\_filename** (*str*) if passed a filename to an HDF5 file containing a orbitize. Result data, MCMC will restart from where it left off.

Written: Jason Wang, Henry Ngo, 2018

#### check\_prior\_support()

Review the positions of all MCMC walkers, to verify that they are supported by the prior space. This function will raise a descriptive ValueError if any positions lie outside prior support. Otherwise, it will return nothing.

(written): Adam Smith, 2021

# chop\_chains(burn, trim=0)

Permanently removes steps from beginning (and/or end) of chains from the Results object. Also updates *curr\_pos* if steps are removed from the end of the chain.

#### **Parameters**

- **burn** (*int*) The number of steps to remove from the beginning of the chains
- **trim** (*int*) The number of steps to remove from the end of the chians (optional)

**Warning:** Does not update bookkeeping arrays within *MCMC* sampler object.

(written): Henry Ngo, 2019

**examine\_chains**(param\_list=None, walker\_list=None, n\_walkers=None, step\_range=None, transparency=1)

Plots position of walkers at each step from Results object. Returns list of figures, one per parameter :param param\_list: List of strings of parameters to plot (e.g. "sma1")

If None (default), all parameters are plotted

#### **Parameters**

- walker\_list List or array of walker numbers to plot If None (default), all walkers are
  plotted
- **n\_walkers** (*int*) Randomly select *n\_walkers* to plot Overrides walker\_list if this is set If None (default), walkers selected as per *walker\_list*
- **step\_range** (*array or tuple*) Start and end values of step numbers to plot If None (default), all the steps are plotted
- **transparency** (*int or float*) Determines visibility of the plotted function If 1 (default) results plot at 100% opacity

#### Returns

Walker position plot for each parameter selected

#### Return type

List of matplotlib.pyplot.Figure objects

(written): Henry Ngo, 2019

**run\_sampler**(total\_orbits, burn\_steps=0, thin=1, examine\_chains=False, output\_filename=None, periodic\_save\_freq=None)

Runs PT MCMC sampler. Results are stored in self.chain and self.lnlikes. Results also added to orbitize.results.Results object (self.results)

**Note:** Can be run multiple times if you want to pause and inspect things. Each call will continue from the end state of the last execution.

#### **Parameters**

- **total\_orbits** (*int*) total number of accepted possible orbits that are desired. This equals num\_steps\_per\_walker x num\_walkers
- **burn\_steps** (*int*) optional paramter to tell sampler to discard certain number of steps at the beginning
- thin (int) factor to thin the steps of each walker by to remove correlations in the walker steps
- **examine\_chains** (*boolean*) Displays plots of walkers at each step by running *examine\_chains* after *total\_orbits* sampled.
- output\_filename (str) Optional filepath for where results file can be saved.
- **periodic\_save\_freq** (*int*) Optionally, save the current results into output\_filename every nth step while running, where n is value passed into this variable.

#### **Returns**

the sampler used to run the MCMC

#### **Return type**

emcee.sampler object

### validate\_xyz\_positions()

If using the XYZ basis, walkers might be initialized in an invalid region of parameter space. This function fixes that by replacing invalid positions by new randomly generated positions until all are valid.

#### **Parameters**

- system (system.System) system.System object
- **like** (*string*) name of likelihood function in lnlike.py
- **custom\_lnlike** (*func*) ability to include an addition custom likelihood function in the fit. The function looks like clnlikes = custon\_lnlike(params) where params is a RxM array of fitting parameters, where R is the number of orbital parameters (can be passed in system.compute\_model()), and M is the number of orbits we need model predictions for. It returns clnlikes which is an array of length M, or it can be a single float if M = 1.

Written: Isabel Angelo, Sarah Blunt, Logan Pearce, 2018

#### prepare\_samples(num\_samples)

Prepare some orbits for rejection sampling. This draws random orbits from priors, and performs scale & rotate.

#### **Parameters**

**num\_samples** (*int*) – number of orbits to draw and scale & rotate for OFTI to run rejection sampling on

#### Returns

array of prepared samples. The first dimension has size of num\_samples. This should be passed into OFTI.reject()

#### **Return type**

np.array

# reject(samples)

Runs rejection sampling on some prepared samples.

#### **Parameters**

**samples** (*np.array*) – array of prepared samples. The first dimension has size num\_samples. This should be the output of prepare\_samples().

#### Returns

np.array: a subset of samples that are accepted based on the data.

np.array: the log likelihood values of the accepted orbits.

# Return type

tuple

run\_sampler(total\_orbits, num\_samples=10000, num\_cores=None, OFTI\_warning=60.0)

Runs OFTI in parallel on multiple cores until we get the number of total accepted orbits we want.

#### **Parameters**

- total\_orbits (int) total number of accepted orbits desired by user
- num\_samples (int) number of orbits to prepare for OFTI to run rejection sampling on. Defaults to 10000.
- num\_cores (int) the number of cores to run OFTI on. Defaults to number of cores availabe.
- **OFTI\_warning** (*float*) if OFTI doesn't accept a single orbit before this amount of time (in seconds), print a warning suggesting to try MCMC. If None, don't print a warning.

#### **Returns**

array of accepted orbits. Size: total\_orbits.

# Return type

np.array

Written by: Vighnesh Nagpal(2019)

class orbitize.sampler.Sampler(system, like='chi2\_lnlike', custom\_lnlike=None, chi2\_type='standard')

Abstract base class for sampler objects. All sampler objects should inherit from this class.

Written: Sarah Blunt, 2018

# 2.5.13 System

A class to store information about a system (data & priors) and calculate model predictions given a set of orbital parameters.

#### **Parameters**

- num\_secondary\_bodies (int) number of secondary bodies in the system. Should be at least 1.
- data\_table (astropy.table.Table) output from orbitize.read\_input. read\_file()
- **stellar\_or\_system\_mass** (*float*) mass of the primary star (if fitting for dynamical masses of both components) or total system mass (if fitting using relative astrometry only) [M\_sol]
- plx (float) mean parallax of the system, in mas
- mass\_err (float, optional) uncertainty on stellar\_or\_system\_mass, in M\_sol
- plx\_err (float, optional) uncertainty on plx, in mas
- restrict\_angle\_ranges (bool, optional) if True, restrict the ranges of the position angle of nodes to [0,180) to get rid of symmetric double-peaks for imaging-only datasets.
- tau\_ref\_epoch (float, optional) reference epoch for defining tau (MJD). Default is 58849 (Jan 1, 2020).
- **fit\_secondary\_mass** (*bool*, *optional*) if True, include the dynamical mass of the orbiting body as a fitted parameter. If this is set to False, stellar\_or\_system\_mass is taken to be the total mass of the system. (default: False)
- hipparcos\_IAD (orbitize.hipparcos.HipparcosLogProb) an object containing information & precomputed values relevant to Hipparcos IAD fitting. See hipparcos.py for more details.
- **gaia** (orbitize.gaia.GaiaLogProb) an object containing information & precomputed values relevant to Gaia astrometrry fitting. See gaia.py for more details.
- **fitting\_basis** (*str*) the name of the class corresponding to the fitting basis to be used. See basis.py for a list of implemented fitting bases.
- use\_rebound (bool) if True, use an n-body backend solver instead of a Keplerian solver.

Priors are initialized as a list of orbitize.priors.Prior objects and stored in the variable System.sys\_priors. You should initialize this class, then overwrite priors you wish to customize. You can use the System.param\_idx attribute to figure out which indices correspond to which fitting parameters. See the "changing priors" tutorial for more detail.

Written: Sarah Blunt, Henry Ngo, Jason Wang, 2018

compute\_all\_orbits(params\_arr, epochs=None, comp\_rebound=False)

Calls orbitize.kepler.calc\_orbit and optionally accounts for multi-body interactions. Also computes total quantities like RV (without jitter/gamma)

- params\_arr (np.array of float) RxM array of fitting parameters, where R is the number of parameters being fit, and M is the number of orbits we need model predictions for. Must be in the same order documented in System() above. If M=1, this can be a 1d array.
- epochs (np.array of float) epochs (in mjd) at which to compute orbit predictions.
- **comp\_rebound** (*bool*, *optional*) A secondary optional input for use of N-body solver Rebound; by default, this will be set to false and a Kepler solver will be used instead.

#### Returns

raoff (np.array of float): N\_epochs x N\_bodies x N\_orbits array of RA offsets from barycenter at each epoch.

**decoff (np.array of float):** N\_epochs x N\_bodies x N\_orbits array of Dec offsets from barycenter at each epoch.

vz (np.array of float): N\_epochs x N\_bodies x N\_orbits array of radial velocities at each epoch.

# Return type

tuple

compute\_model(params\_arr, use\_rebound=False)

Compute model predictions for an array of fitting parameters. Calls the above compute\_all\_orbits() function, adds jitter/gamma to RV measurements, and propagates these predictions to a model array that can be subtracted from a data array to compute chi2.

#### **Parameters**

- params\_arr (np.array of float) RxM array of fitting parameters, where R is the number of parameters being fit, and M is the number of orbits we need model predictions for. Must be in the same order documented in System() above. If M=1, this can be a 1d array.
- use\_rebound (bool, optional) A secondary optional input for use of N-body solver Rebound; by default, this will be set to false and a Kepler solver will be used instead.

#### Returns

np.array of float: Nobsx2xM array model predictions. If M=1, this is a 2d array, otherwise it is a 3d array.

**np.array of float:** Nobsx2xM array jitter predictions. If M=1, this is a 2d array, otherwise it is a 3d array.

#### Return type

tuple of

# convert\_data\_table\_radec2seppa(body\_num=1)

Converts rows of self.data\_table given in radec to seppa. Note that self.input\_table remains unchanged.

### **Parameters**

**body\_num** (int) – which object to convert (1 = first planet)

save(hf)

Saves the current object to an hdf5 file

#### **Parameters**

**hf** (h5py.\_h1.files.File) – a currently open hdf5 file in which to save the object.

orbitize.system.radec2seppa(ra, dec, mod180=False)

Convenience function for converting from right ascension/declination to separation/ position angle.

#### **Parameters**

- ra (np. array of float) array of RA values, in mas
- dec (np.array of float) array of Dec values, in mas
- mod180 (Bool) if True, output PA values will be given in range [180, 540) (useful for plotting short arcs with PAs that cross 360 during observations) (default: False)

#### Returns

(separation [mas], position angle [deg])

# Return type

tuple of float

orbitize.system.seppa2radec(sep, pa)

Convenience function to convert sep/pa to ra/dec

#### **Parameters**

- **sep** (*np.array of float*) array of separation in mas
- pa (np.array of float) array of position angles in degrees

#### Returns

(ra [mas], dec [mas])

#### Return type

tuple

orbitize.system.transform\_errors(x1, x2, x1\_err, x2\_err, x12\_corr, transform\_func, nsamps=100000)

Transform errors and covariances from one basis to another using a Monte Carlo apporach

- **x1** (float) planet location in first coordinate (e.g., RA, sep) before transformation
- x2 (float) planet location in the second coordinate (e.g., Dec, PA) before transformation)
- x1\_err (float) error in x1
- **x2\_err** (*float*) error in x2
- **x12\_corr** (*float*) correlation between x1 and x2
- transform\_func (function) function that transforms between (x1, x2) and (x1p, x2p) (the transformed coordinates). The function signature should look like: x1p,  $x2p = transform\_func(x1, x2)$
- **nsamps** number of samples to draw more the Monte Carlo approach. More is slower but more accurate.

# 2.6 orbitize! Manual

#### 2.6.1 Intro to orbitize!

orbitize! hinges on the two-body problem, which describes the paths of two bodies gravitationally bound to each other as a function of time, given parameters determining the position and velocity of both objects at a particular epoch. There are many basis sets (orbital bases) that can be used to describe an orbit, which can then be solved using Kepler's equation, but first it is important to be explicit about coordinate systems.

**Note:** For an interactive visualization to define and help users understand our coordinate system, you can check out this GitHub tutorial.

There is also a YouTube video with use and explanation of the coordinate system by Sarah Blunt.

In its "standard" mode, orbitize! assumes that the user only has relative astrometric data to fit. In the orbitize! coordinate system, relative R.A. and declination can be expressed as the following functions of orbital parameters

$$\Delta R.A. = \pi a (1 - e \cos E) \left[\cos^2 \frac{i}{2} \sin(f + \omega_p + \Omega) - \sin^2 \frac{i}{2} \sin(f + \omega_p - \Omega)\right]$$
  
$$\Delta decl. = \pi a (1 - e \cos E) \left[\cos^2 \frac{i}{2} \cos(f + \omega_p + \Omega) + \sin^2 \frac{i}{2} \cos(f + \omega_p - \Omega)\right]$$

where , ,  $\omega_p$ , , and are orbital parameters, and is the system parallax. f is the true anomaly, and E is the eccentric anomaly, which are related to elapsed time through Kepler's equation and Kepler's third law

$$M = 2\pi \left(\frac{t}{P} - (\tau - \tau_{ref})\right)$$
$$\left(\frac{P}{yr}\right)^2 = \left(\frac{a}{au}\right)^3 \left(\frac{M_{\odot}}{M_{tot}}\right)$$
$$M = E - esinE$$
$$f = 2tan^{-1} \left[\sqrt{\frac{1+e}{1-e}}tan\frac{E}{2}\right]$$

orbitize! employs two Kepler solvers to convert between mean and eccentric anomaly: one that is efficient for the highest eccentricities, and Newton's method, which in other cases is more efficient for solving for the average orbit. See Blunt et al. (2020) for more detail.

From scrutinizing the above sets of equations, one may observe a few important degeneracies:

- 1. Individual component masses do not show up anywhere in this equation set.
- 2. The degeneracy between semimajor axis, total mass tot, and parallax. If we just had relative astrometric measurements and no external knowledge of the system parallax, we would not be able to distinguish between a system that has larger distance and larger semimajor axis (and therefore larger total mass, assuming a fixed period) from a system that has smaller distance, smaller semimajor axis, and smaller total mass.
- 3. The argument of periastron  $\omega_p$  and the position angle of nodes . The above defined R.A. and decl. functions are invariant to the transformation:

$$\omega_p' = \omega_p + \pi$$
$$\Omega' = \Omega - \pi$$

which creates a 180° degeneracy between particular values of  $\omega_p$  and , and a characteristic "double-peaked" structure in marginalized 1D posteriors of these parameters.

Solutions to breaking degeneracies 1 and 3 can be found in the next section.

# 2.6.2 Using Radial Velocities

In the orbitize! coordinate system, and relative to the system barycenter, the radial velocity of the planet due to the gravitational influence of the star is:

$$rv_p(f) = \sqrt{\frac{G}{(1 - e^2)}} M_* sini(M_{tot})^{-1/2} a^{-1/2} (cos(\omega_p + f) + ecos\omega_p)$$
$$rv_*(f) = \sqrt{\frac{G}{(1 - e^2)}} M_p sini(M_{tot})^{-1/2} a^{-1/2} (cos(\omega_* + f) + ecos\omega_*)$$

where is the argument of periastron of the star's orbit, which is equal to + 180°.

In these equations, the individual component masses m and m enter. This means radial velocity measurements break the total mass degeneracy and enable measurements of individual component masses ("dynamical" masses). However it is crucial to keep in mind that radial velocities of a planet do not enable dynamical mass measurements of the planet itself, but of the star. Radial velocity measurements also break the / degeneracy discussed in the previous section, uniquely orienting the orbit in 3D space.

orbitize! can perform joint fits of RV and astrometric data in two different ways, which have complementary applications.

The first method is automatically triggered when an orbitize! user inputs radial velocity data. orbitize! automatically parses the data, sets up an appropriate model, then runs the user's Bayesian computation algorithm of choice to jointly constrain all free parameters in the fit. orbitize! can handle both primary and secondary RVs, and fits for the appropriate dynamical masses when RVs are present; when primary RVs are included, orbitize! fits for the dynamical masses of secondary objects, and vice versa. Instrumental nuisance parameters (RV zeropoint offset, , and white noise jitter, ) for each RV instrument are also included as additional free parameters in the fit if the user specifies different instrument names in the data file.

The second method of jointly fitting RV and astrometric data in orbitize! separates out the fitting of radial velocities and astrometry, enabling a user to fit "one at a time," and combine the results in a Bayesian framework. First, a user performs a fit to just the radial velocity data using, for example, radvel (but can be any radial velocity orbit-fitting code). The user then feeds the numerical posterior samples into orbitize! through the orbitize.priors.KDEPrior object. This prior creates a representation of the prior using kernel density estimation (kernel density estimation), which can then be used to generate random prior samples or compute the prior probability of a sample orbit. Importantly, this prior preserves covariances between input parameters, allowing orbitize! to use an accurate representation of the RV posterior to constrain the fit. This method can be referred to as the "posteriors as priors" method, since posteriors output from a RV fitting code are, through KDE sampling, being applied as priors in orbitize!

More coming soon!

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# THREE

# **CHANGELOG:**

### 3.0.0 (TBD)

- discuss MCMC autocorrelation in MCMC tutorial (@michaelkmpoon)
- add time warning if OFTI doesn't accept an orbit in first 60 s (@michaelkmpoon)

### 2.2.2 (2023-06-30)

• tests now overwrite any generated text files (@sblunt)

### 2.2.1 (2023-06-28)

• tau\_to\_tp function now accepts array of after\_date (@tomasstolker/@semaphoreP)

# 2.2.0 (2023-06-21)

- set up new CI system using GH actions (@sblunt)
- removed radvel as dependency, and moved radvel\_utils subpackage to new dir (@sblunt). This is a breaking change for users of orbitize.radvel\_utils.

# 2.1.4 (2023-06-20)

- unit tests hotfixes (@semaphoreP)
- use forked ptemcee (@sblunt)

# 2.1.3 (2023-02-07)

• Compatibility with numpy v1.24 (issue #330 and #331; @tomasstolker)

#### 2.1.2 (2022-08-31)

• Bugfix for saving/loading fits using IAD (issue #324; @sblunt)

# 2.1.1 (2022-05-24)

• Hotfix for one of the log-chi2 unit tests (@sblunt)

#### 2.1.0 (2022-05-24)

• Added a (more numerically stable) log-chi2 option for calculating likelihood (@Mireya-A and @lhirsch238)

# 2.0.1 (2022-04-22)

- Addressed plotting bugs: issues #316/#309, #314, #311 (@semaphoreP)
- Made Gaia module runnable without internet and added some Gaia/Hipparcos unit tests (@sblunt)

### 2.0.0 (2021-10-13)

This is the official release of orbitize! version 2.

Big changes:

- Fit Gaia positions (@sblunt)
- New plotting module & API (@sblunt)
- Relative planet RVs now officially supported & tested (@sblunt)
- GPU Kepler solver (@devincody)
- RV end-to-end test added (@vighnesh-nagpal)

#### Small changes:

- Hipparcos calculation bugfix (@sblunt)
- v1 results backwards compatibility bugfix (@sblunt)
- windows install docs update (@sblunt
- basis bugfix with new API (@TirthDS, @sblunt)
- handle Hipparcos 2021 data format (@sblunt)
- clarify API on mtot/mstar (@lhirsch238, @sblunt)

#### 2.0b1 (2021-09-03)

This is the beta release of orbitize! version 2.

#### Big changes:

- N-body Kepler solver backend! (@sofiacovarrubias)
- Fitting in XYZ orbital basis! (@rferrerc)
- API for fitting in arbitrary new orbital bases! (@TirthDS)
- compute\_all\_orbits separated out, streamlining stellar astrometry & RV calculations (@sblunt)
- Hip IAD! (@sblunt)
- param\_idx now used everywhere under the hood (system parsing updated) (@sblunt)
- KDE prior added (inspiration=training on RV fits) (@jorgellop)

### Small changes:

- HD 4747 rv data file fix for the RV tutorial (@lhirsch238)
- Add check prior support to sampler.MCMC (@adj-smith)
- Update example generation code in MCMC v OFTI tutorial (@semaphoreP)
- Fixed plotting bug (issue #243) (@TirthDS)
- Expand FAQ section (@semaphoreP)
- use astropy tables in results (@semaphoreP)
- Expand converge section of MCMC tutorial (@semaphoreP)
- Deprecated functions and deprecation warnings officially removed (@semaphoreP)
- Fix logic in setting of track\_planet\_perturbs (@sblunt)
- Fix plotting error if orbital periods are > 1e9 days (@sblunt)
- Add method for printing results of a fit (@sblunt)

# 1.16.1 (2021-06-27)

• Fixed chop\_chains() function to copy original data over when updating Results object (@TirthDS)

#### 1.16.0 (2021-06-23)

- User-defined prior on PAN were not being applied if OFTI is used; fixed (@sblunt)
- Dates in HD 4747 data file were incorrect; fixed (@lhirsch238)

#### 1.15.5 (2021-07-20)

- Addressed issue #177, giving *Results* and *Sampler* classes a parameter label array (@sblunt)
- Fixed a bug that was causing RA/Dec data points to display wrong in orbit plots (@sblunt)

# 1.15.4 (2021-06-18)

• Bugfix for issue #234 (@semaphoreP, @adj-smith)

# 1.15.3 (2021-06-07)

Add codeastro mode to pytest that prints out a SECRET CODE if tests pass omgomg (@semaphoreP)

# 1.15.2 (2021-05-11)

Fixed backwards-compatibility bug with version numbers and saving/loading (@semaphoreP, @wbalmer)

#### 1.15.1 (2021-03-29)

- Fixed bug where users with Results objects from v<14.0 couldn't load using v>=14.0 (@semaphoreP, @wbalmer)
- Fixed order of Axes objects in Advanced Plotting tutorial (@wbalmer, @sblunt)

#### 1.15.0 (2021-02-23)

Handle covariances in input astrometry (@semaphoreP)

#### 1.14.0 (2021-02-12)

- Version number now saved in results object (@hgallamore)
- Joint RV+astrometry fits can now handle different RV instruments! (@vighnesh-nagpal, @Rob685, @lhirsch238)
- New "FAQ" section added to docs (@semaphoreP)
- Bugfix for multiplanet code (@semaphoreP) introduced in PR #192
- now you can pass a preexisting Figure object into results.plot\_orbit (@sblunt)
- colorbar label is now "Epoch [year]" (@sblunt)
- corner plot maker can now handle fixed parameters without crashing (@sblunt)

### 1.13.1 (2021-01-25)

- compute\_sep in radvel\_utils submodule now returns mp (@sblunt)
- astropy.\_erfa was deprecated (now in separate package). Dependencies updated. (@sblunt)

# 1.13.0 (2020-11-8)

- Added radvel-utils submodule which allows users to calculate projected separation posteriors given RadVel chains (@sblunt)
- Fixed a total mass/primary mass mixup bug that was causing problems for equal-mass binary RV+astrometry joint fits (@sblunt)
- Bugfix for multiplanet perturbation approximation: now only account for inner planets only when computing perturbations (@semaphoreP)

### 1.12.1 (2020-9-6)

- tau\_ref\_epoch is now set to Jan 1, 2020 throughout the code (@semaphoreP)
- restrict\_angle\_ranges keyword now works as expected for OFTI (@sblunt)

# 1.12.0 (2020-8-28)

• Compatibility with emcee>=3 (@sblunt)

#### 1.11.3 (2020-8-20)

- Save results section of OFTI tutorial now current (@rferrerc)
- Modifying MCMC initial positions tutorial documentation now uses correct orbital elements (@rferrerc)

# 1.11.2 (2020-8-10)

- Added transparency option for plotting MCMC chains (@sofiacovarrubias)
- Removed some redundant code (@MissingBrainException)

# 1.11.1 (2020-6-11)

• Fixed a string formatting bug causing corner plots to fail for RV+astrometry fits

#### 1.11.0 (2020-4-14)

- Multiplanet support!
- Changes to directory structure of sample data files
- Fixed a bug that was causing corner plots to fail on loaded results objects

### 1.10.0 (2020-3-6)

- Joint RV + relative astrometry fitting capabilities!
- · New tutorial added

#### 1.9.0 (2020-1-24)

- Require astropy>=4
- Minor documentation upgrades
- This is the first Python 2 noncompliant version

# 1.8.0 (2020-1-24)

- Bugfixes related to numpy and astropy upgrades
- This is the last version that will support Python 2

# 1.7.0 (2019-11-10)

- Default corner plots now display angles in degrees instead of radians
- Add a keyword for plotting orbits that cross PA=360

# 1.6.0 (2019-10-1)

- Mikkola solver now implemented in C-Kepler solver
- Fixed a bug with parallel processing for OFTI
- Added orbit vizualisation jupyter nb show-me-the-orbit to docs/tutorials
- · New methods for viewing/chopping MCMC chains
- Require emcee<3 for now

### 1.5.0 (2019-9-9)

- Parallel processing for OFTI.
- Fixed a bug converting errors in RA/Dec to sep/PA in OFTI.
- OFTI and MCMC now both return likelihood, whereas before one returned posterior.
- Updated logic for restricting Omega and omega bounds.

#### 1.4.0 (2019-7-15)

- API change to lay the groundwork for dynamical mass calculation.
- JeffreysPrior -> LogUniformPrior
- · New tutorials.
- Added some informative error messages for input tables.
- · Bugfixes.

# 1.3.1 (2019-6-19)

• Bugfix for RA/Dec inputs to the OFTI sampler (Issue #108).

# 1.3.0 (2019-6-4)

- Add ability to customize date of tau definition.
- Sampler now saves choice of tau reference with results.
- Default tau value is now Jan 1, 2020.
- · Small bugfixes.

# 1.2.0 (2019-3-21)

- Remove unnecessary astropy date warnings.
- · Add custom likelihood function.
- Add progress bar for ptemcee sampler.
- Add customizable color axis for orbit plots.
- · Small bugfixes.

# 1.1.0 (2019-1-6)

- Add sep/PA panels to orbit plot.
- GaussianPrior now operates on only positive numbers by default.

# 1.0.2 (2018-12-4)

- Expand input reading functionality.
- Bugfixes for MCMC.

# 1.0.1 (2018-11-20)

· Bugfix for building on CentOS machines.

# 1.0.0 (2018-10-30)

Initial release.

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